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ESTIMATING HORIZONTAL DRAIN DESIGN BY THE FINITE-ELEMENT AND FINITE-DIFFERENCE METHODS

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ABSTRACT

The presence of water is one of the most critical factors contributing to the instability of tailings embankments. Nonexistent or inefficient drainage facilities usually result in a high phreatic surface that can cause the embankment to fail. One possible solution, especially for remedial situations, is the installation of horizontal drains.

An investigation into the effects of horizontal drains on the phreatic surface of tailings embankments was conducted by the Bureau of Mines. A laboratory model was constructed to correlate the effects of various drain spacing and length combinations with results from computer codes. A three-dimensional finite-element computer code and a two-dimensional finite-difference computer code were used. Two existing tailings embankments were modeled and potentiometric data were compared to the computer results.

The two-dimensional finite-difference code and the three-dimensional finite-element code produced nearly the same location for the phreatic surface between drains. The location of the phreatic surface from the codes was slightly above the phreatic surface of one field application and followed the trend of the other closely. The phreatic surface location of the laboratory model was slightly above that of the computer-generated phreatic surface. The distance between the phreatic surfaces of these models became larger as drain length increased or drain spacing decreased.

Dimensionless graphs were constructed for estimating phreatic surface location for embankments with horizontal drains.

INTRODUCTION

The height of the phreatic surface plays a critical role in the determination of the factor of safety in soils embankments [11]. If the height of the phreatic surface can be lowered, the factor of safety will increase dramatically. The design engineer is faced with the problem of determining what dimensions (spacing and length of drain) are necessary in order to reduce the phreatic surface adequately to ensure an acceptable factor of safety. Various analytical techniques are available for solving this problem.

The use of two-dimensional techniques to determine seepage characteristics in embankments without drains has become a common engineering practice [3-6, 12-17]. The effects of toe drains or blanket drains also can be modeled with two-dimensional computer codes, provided all cross sections of the embankment are the same. The design of horizontal drains, however, is a three-dimensional problem, and analysis can be time-consuming. Three-dimensional finite-element meshes require a considerable amount of time to construct, and three-dimensional finite-element computer codes often require large computer time and space requirements due to the large number of unknown values that must be computed. It is the intent of this paper to compare results from two-dimensional and three-dimensional computer codes and present results from a parametric study involving embankment geometry, drain length, and drain spacing in a graphic format.

The authors wish to thank Fred Tracy, supervisory computer scientist, U.S. Army Engineer Division, U.S. Army Engineer Waterways Experiment Station, Vicksburg, MS, for assistance in using the three-dimensional finite-element computer program; and John P. Sanders, plant superintendent, Union Carbide Corporation, Metals Division, Hot Springs, AR, for providing potentiometric data and data on embankment geometry.

LABORATORY MODEL

The 2.44- by 1.83-m (96- by 72-in) model tank was constructed of 2.65-mm (12-ga) cold-rolled steel with welded seams and was mounted on a structural steel framework on wheels. The inside walls and floor were coated with latex-base paint. While still wet, the surfaces were sprinkled with 16-mesh (1.18-mm openings) sand to prevent flow channels from developing along the sides and bottom of the tank.

Clusters of piezometers of increasing height were installed, as shown in figures 1 and 2. They were constructed of 3.97-mm-ID (5/32-in-ID) copper tubing with 140-mesh (0.106-mm openings) screen soldered to the top of each tube. The screen was covered with filter cloth. Plexiglas[1] viewing tubes connected to the piezometers, were mounted to the side of the model. A blue dye was injected into each tube for ease of reading.

Some of the filters on the piezometers became clogged during initial tests, so open-well piezometers were subsequently installed. They were constructed of 1.02-mm-ID (1/4-in-ID) perforated brass tubing with 140-mesh (0.106-mm openings) screen soldered over the perforations and bottom end of each tube. The tubes were installed at the same location as the piezometer clusters (fig. 2) excluding the first row near the toe of the embankment.

[1]Reference to specific equipment, trade names, or manufacturers does not imply endorsement by the Bureau of Mines.

The horizontal drains were constructed of 1.02-mm-ID (1/4-in-ID) brass tubing with fifty-nine 3.18-mm holes drilled per meter (eighteen 1/8-in holes drilled per foot). This results in 2.3 pct open drain per unit length. By comparison, a typical 5.08-cm (2-in) polyvinyl chloride (PVC) slotted pipe having three 0.40-mm (1/64-in) slots around the circumference spaced 2.54 cm (1-in) apart along the pipe has approximately 1.4 pct open drain per unit length. Screen (140 mesh) was soldered over the holes to prevent the drains from clogging. The drains were attached to sections of rods which were threaded through holes in Teflon flourocarbon polymer brackets so they could be pulled through the embankment to acheive various spacing and length combinations. Hydraulic seals were used to prevent leakage where the rods were pulled through the back of the tank. The drains and piezometer tubes were spaced 0.23 m (9 in) apart across the width of the tank so piezometer readings could be taken at each drain location.

The embankment was constructed of silica sand having a permeability (k) of 3 x 10^{-4} cm/sec determined from a constant head permeability test. Dry density of the material was 13,085 N/M^3 (83.3 pcf). The grain-size distribution is shown in figure 3, and the Standard Proctor test results are shown in figure 4. Drained direct shear tests yielded an angle of internal friction of 37° and a cohesion of 62.04 kPa (9 psi). The downstream slope was 2:1, and the upstream slope was 1:1 (fig. 1). The embankment was compacted by hand using a 1.91-cm (3/4-in) pipe attached to a 2.54-cm (1-in) thick, 15.24-cm (6-in) diam steel plate.

The first trial embankment experienced progressive failure due to erosion when subjected to a headwater height of 49.02 cm (19.3 in), so a 25.4-cm (10-in) toe drain composed of coarse sand (k = 3.75 x 10^{-2} cm/sec) was installed to increase stability. All subsequent tests were run with the toe drain. A constant upstream head was maintained during the tests by a float valve.

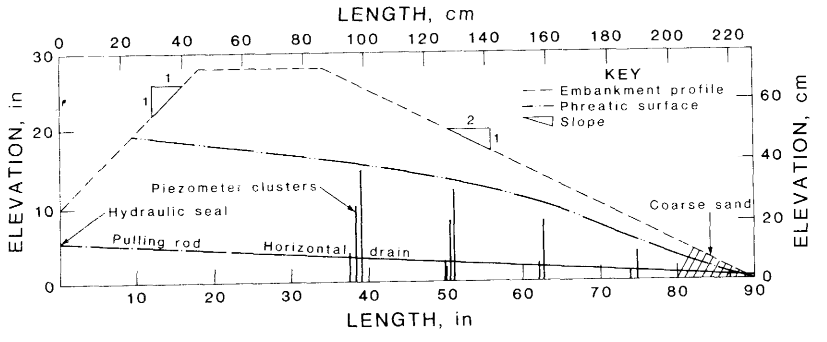

FIGURE 1. Cross section of laboratory embankment model.

FINITE-DIFFERENCE CODE

The governing equations approximated by the finite-difference computer program [15] are based on Darcy's law in two dimensions:

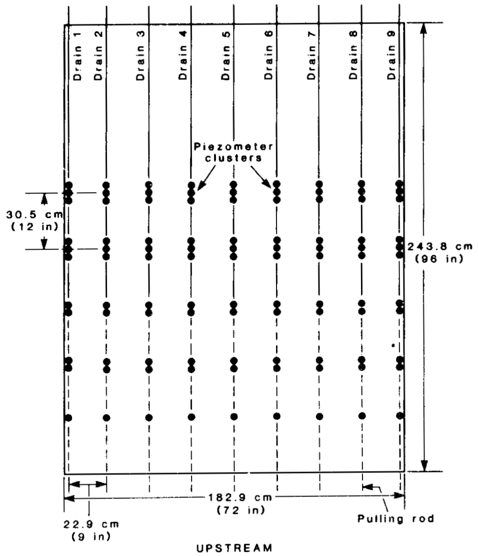

FIGURE 2. Plan view of laboratory model.

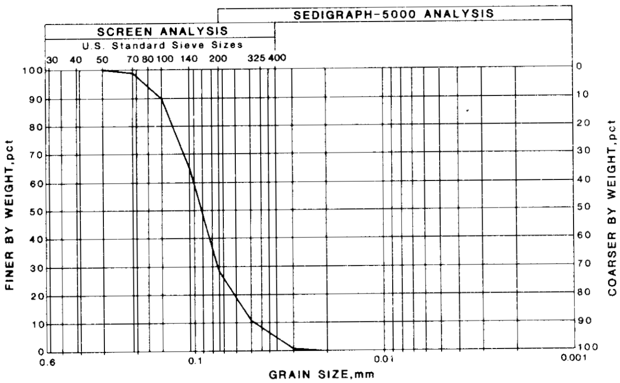

FIGURE 3. Grain-size curve of sand sample from laboratory model.

$$q_x = -k\frac{\partial h}{\partial x}, \qquad (1)$$

$$q_y = -k\frac{\partial h}{\partial y}, \qquad (2)$$

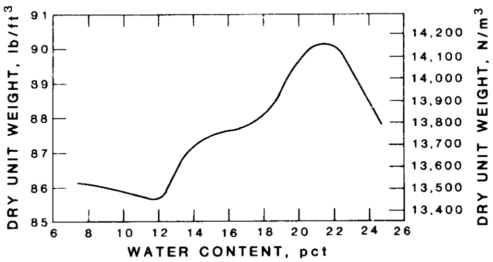

FIGURE 4. Moisture-density curve for sand used in laboratory model.

where q_x = Darcy velocity in x direction,
q_y = Darcy velocity in y direction,
k = permeability of the soil, $k=k(x,y)$,
and h = total head.

The continuity equation in two dimensions is:

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0. \tag{3}$$

Substituting equations 1 and 2 into equation 3 yields:

$$\frac{\partial}{\partial x}\left(-k\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(-k\frac{\partial h}{\partial y}\right) = 0. \tag{4}$$

If the soil (or tailings material) is assumed to be homogeneous and isotropic,[2] then k is independent of x and y, and equation 4 becomes Laplace's equation,

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0. \tag{5}$$

The flow region was modeled using the plan view (fig. 2). Boundary conditions are specified in figure 5.

This model does not include the z component of velocity. This condition is known as the Dupuit assumption. The validity of the assumption is evaluated analytically by Murray and Monkmeyer [9]. In general, best results using the Dupuit assumption are achieved for situations where the slope of the phreatic surface is relatively flat (10:1). It will be shown later that the phreatic surface between drains from two-dimensional analysis compares favorably to the results from a three-dimensional analysis.

[2]Horizontal permeability is often greater than vertical permeability in hydraulically placed tailings material [6]. If this condition exists, it is likely that the phreatic surface will be higher than the phreatic surface in an embankment with isotropic properties.

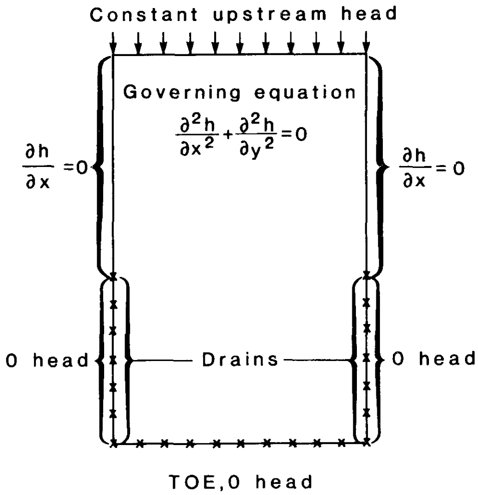

FIGURE 5. Boundary conditions for finite-difference model.

FINITE-ELEMENT CODE

The three-dimensional finite-element program (program number 704-F3-R0218) was developed by the U.S. Army Waterways Experiment Station [13]. The basic assumptions of the model are -

1. The density of the soil-water complex remains constant, since the compressibility of the soil-water complex is zero.

2. The flow is laminar; hence, Darcy's law holds.

The governing equation is similar to equation 4, only it has a z component:

$$\frac{\partial}{\partial x}\left(-k\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(-k\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(-k\frac{\partial h}{\partial z}\right) = 0. \qquad (6)$$

Since this paper assumes homogeneous soil conditions, k is constant, and equation 6 reduces to Laplace's equation in three dimensions:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0. \qquad (7)$$

A discussion of the solution to equation 7 by the finite-element method has been discussed by Tracy [12].

A cross section of the finite-element mesh used to analyze the laboratory model is shown in figure 6. A total of 448 elements and 648 nodes

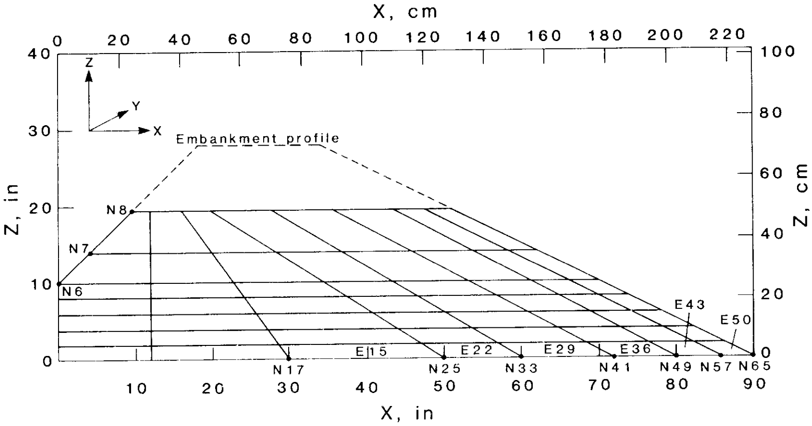

FIGURE 6. Cross section of three-dimensional mesh. (E prefix denotes element; N denotes node)

were used in the simulation. The headwater entered the embankment at nodes 6, 7, and 8 and subsequent nodes in other cross sections in the y direction.

Two methods were used to simulate the drains, each giving the same results. In the first method, elements such as 15, 22, 29, 36, 43, and 50 were assigned a y and z dimension of 1.27 cm (0.5 in) and a very high permeability to represent the drain. The second method used zero pressure as a boundary condition at nodes such as 17, 25, 33, 41, 49, 57, and 65. The boundary conditions for both methods were applied at y = 0 and y = the width of the embankment, so that the phreatic surface was symmetric about a line parallel to and between the drains. Various drain spacings were achieved by changing the width (y dimension) of all the elements. Drain length was changed by altering the element permeability, when the first method was used, or eliminating the boundary condition of zero pressure at a node when the second method was used.

LABORATORY MODEL TESTS

The drains were inserted into the embankment in the sequence listed below (fig. 2). Drain length was measured from the toe of the embankment.

1. No drains inserted.
2. Drains 1 and 9, 0.61 m (2 ft).
3. Drains 1 and 9, 1.22 m (4 ft).
4. Drains 1 and 9, 1.83 m (6 ft).
5. Drains 1 and 9, 1.83 m (6 ft); drain 5, 0.61 m (2 ft).
6. Drains 1 and 9, 1.83 m (6 ft); drain 5, 0.61 m (4 ft).
7. Drains 1, 5, and 9, 1.83 m (6 ft).
8. Drains 1, 5, and 9, 1.83 m (6 ft); drains 3 and 7, 0.61 m (2 ft).
9. Drains 1, 5, and 9, 1.83 m (6 ft); drains 3 and 7, 1.22 m (4 ft).
10. Drains 1, 3, 5, 7, and 9, 1.83 m (6 ft).

11. Drains 1, 3, 5, 7, and 9, 1.83 m (6 ft); drains 2, 4, 6, and 8, 1.22 m (4 ft).
12. All drains inserted 1.83 m (6 ft).

Open-well piezometer readings were taken for each sequence at 1-day intervals. The next drain sequence was not commenced until each piezometer had the same reading for two consecutive days. When all drains were inserted 1.83 m (6 ft) and the phreatic surface had reached steady state, the sequence was executed in reverse order to test the integrity of the model.

The potentiometric data were plotted on three-dimensional graphs for visual interpretation of the effects of the drains. Two example graphs are shown in figures 7 and 8. A one-to-one numerical comparison

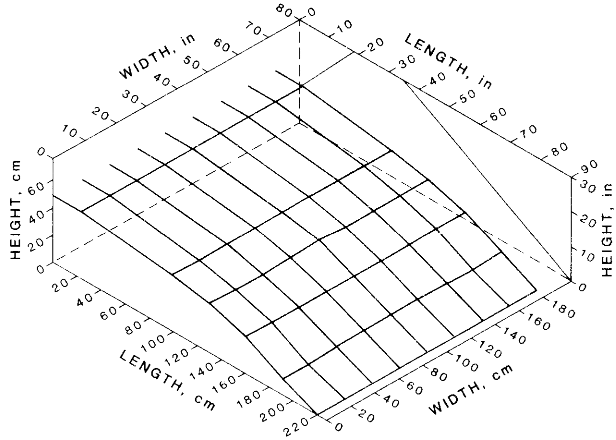

FIGURE 7. Potentiometric data for laboratory embankment model: no drains, headwater = 49.3 cm (19.4 in).

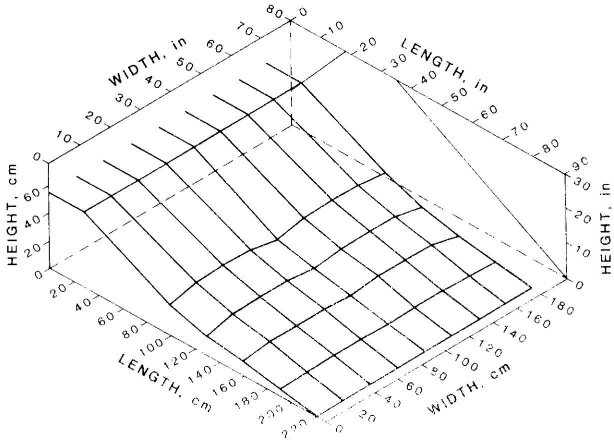

FIGURE 8. Potentiometric data for laboratory embankment model: Drains 1, 5, and 9 ins[illegible], headwater [illegible]

between the two sequences is not valid, because the headwater was increased on the second sequence. However, piezometer response and geometric characteristics were compatible.

LABORATORY MODEL VERSUS COMPUTER MODELS

Only the readings from the open-well piezometers were used to compare the laboratory model to the results from the computer codes. Although some of the piezometers in the clusters became plugged with fines, enough data were available in order to describe the bending of the equipotential lines near the toe of the embankment.

Profiles of the embankment at midpoint, comparing the phreatic surface from the test model to the results of the finite-difference code, were plotted for cases 1 through 4, 7, and 10. Several examples are shown in figures 9 and 10. Maximum variation in phreatic height varied from less than 2.54 cm (1 in) to 5.08 cm (2 in). As drain length increased, or drain spacing decreased, the phreatic surface from the finite-difference code fell below the laboratory model.

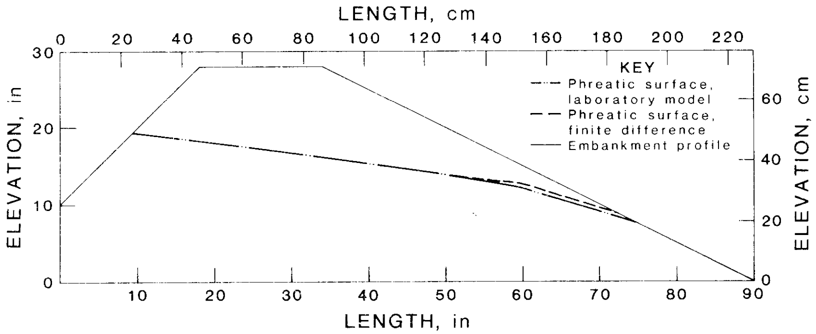

FIGURE 9. Phreatic surface, laboratory model versus finite-difference code, no drains.

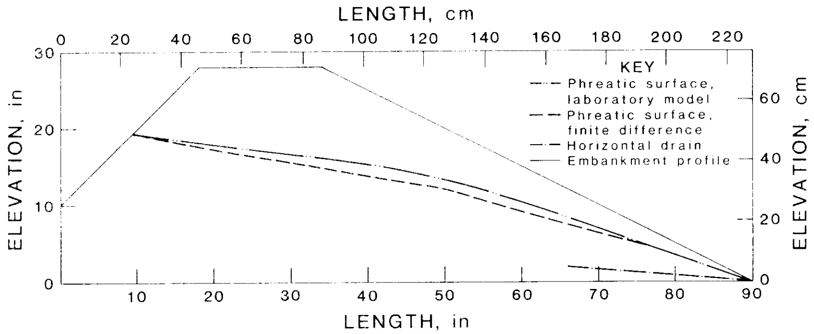

FIGURE 10. Phreatic surface between drains, laboratory model versus finite-difference method: 61.0-cm (24-in) drains, 91.4-cm (36-in) spacing.

Figure 11 compares the phreatic surface between drains computed by the two computer codes for a 1.83-m (6-ft) drain and spacing of 0.41 m (16 in), 0.91 m (36 in), and 1.83 m (72 in). The results indicate that for between drains, the finite-difference code and the three-dimensional finite-element code calculate nearly the same phreatic surface.

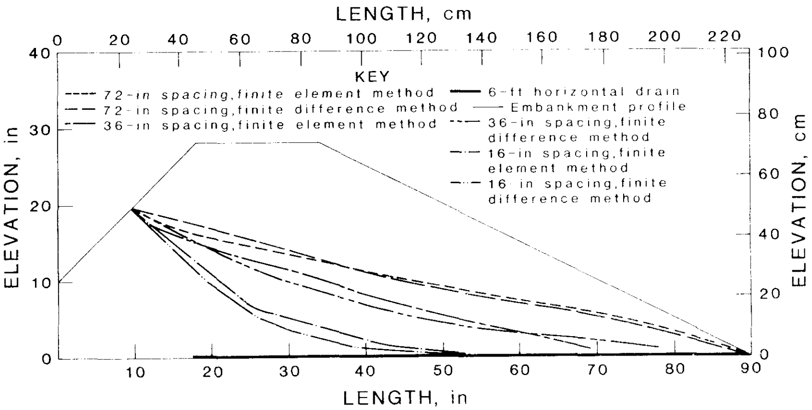

FIGURE 11. Phreatic surface between drains, finite-element method versus finite-difference method.

COMPARISON WITH OTHER LABORATORY DATA

A seepage model experiment conducted by Kenney, Pazin, and Choi [7] was modeled using the three-dimensional finite-element program. The material used to construct the model was glass beads with less than 30 pct retained on the No. 100 sieve (0.150-mm openings); more than 70 pct was retained on the No. 200 sieve (0.075-mm openings). The drains in the model consisted of a 5.08-mm (0.2-in) drain rod, double-wrapped with No. 200-sieve stainless steel mesh, and spot-soldered.

The phreatic surface calculated by the finite-element code is higher than that measured in the laboratory (fig. 12). For the case of no horizontal drains (fig. 13), the maximum difference of approximately 2.21 cm (0.87 in) occurs at x = 40.6 cm (x = 16 in). When horizontal drains 69 cm (27 in) in length having a spacing of 81.3 cm (32 in) are installed, the maximum difference is 2.03 cm (0.8 in) which occurs at x = 81.3 cm (32 in).

COMPUTER CODES VERSUS FIELD DATA

Two tailings embankments were modeled with the two-dimensional finite-difference computer program. Although neither had perfectly parallel horizontal drains nor extensive piezometer readings between drains, each provided a basis for comparing the general effects of the drains to the computer model.

The horizontal drains in the Sohio Western Mining Company's embankment were installed in an array-like pattern of five drains in each array (fig. 14) [8, 16]. The area that was modeled using the finite-

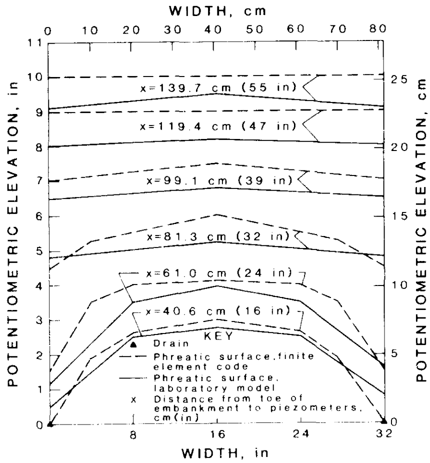

FIGURE 12. Three-dimensional finite-element results versus potentiometric elevations from Kenney, Pazin, and Choi (7): 69-cm (27-in) drains, 81.3-cm (32-in) spacing.

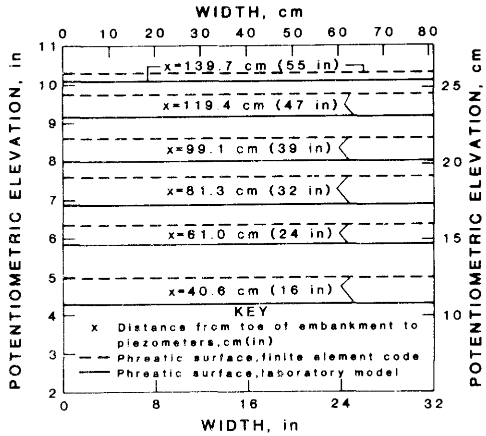

FIGURE 13. Three-dimensional finite-element results versus potentiometric elevations from Kenney, Pazin, and Choi (7): No drains.

difference code is indicated by crosshatching. The boundary conditions and governing equation are as in figure 5, with the additional condition that line AB is a no-flow boundary. This boundary condition is used because the face of the starter dam is clogged with fines [16].

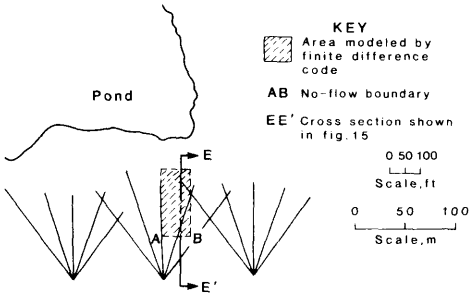

FIGURE 14. Plan view of drain arrays showing area modeled using finite-difference code [8].

A cross-sectional view comparing computer results to piezometric data is shown in figure 15. The modeled phreatic surface oscillates about the measured phreatic surface; however, the general trend is the same.

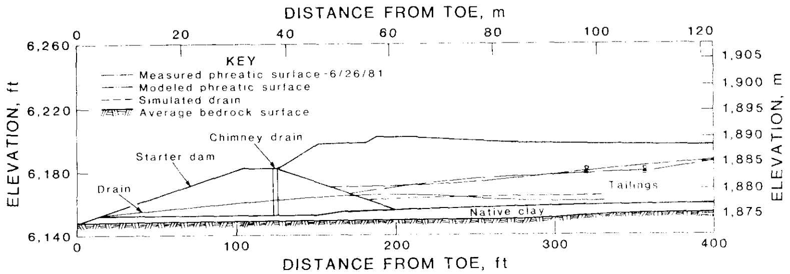

FIGURE 15. Section E-E' from figure 14. Sohio Western Mining Co. tailings embankment [8].

Union Carbide's tailings embankment contains filter pads in addition to horizontal drains. This situation was modeled by assigning the nodes representing the filter pads a value of zero pressure. The filter pads were modeled at a 48.77-m (160-ft) spacing. Field data from only one piezometer were available and ranged from 0 to 0.61 m (0 ft to 2 ft) over a period of 8-1/2 yr. The finite-difference method predicts a value of 2.44 m (8 ft) at the location of this piezometer.

DIMENSIONLESS PHREATIC PROFILES

A parameter study using the two-dimensional code was conducted in order to construct profiles of the phreatic surface midway between drains. Examples are presented in figures 16 through 18 for various drain length and spacing configurations.[3] All parameters are normalized by the distance L--the distance from the headwater to the toe of the embankment. Other parameters are defined as follows:

[3]Steady-state conditions are assumed. The time needed for the drains to reach full efficiency was investigated by Nonveiller [10].

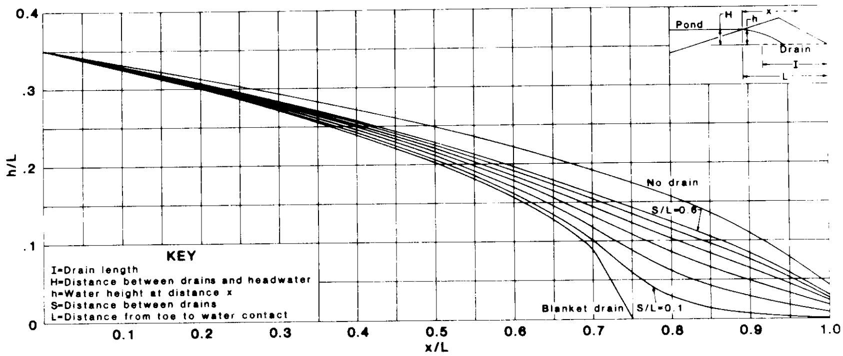

FIGURE 16. Dimensionless phreatic profiles, H/L = 0.35, I/L = 0.25.

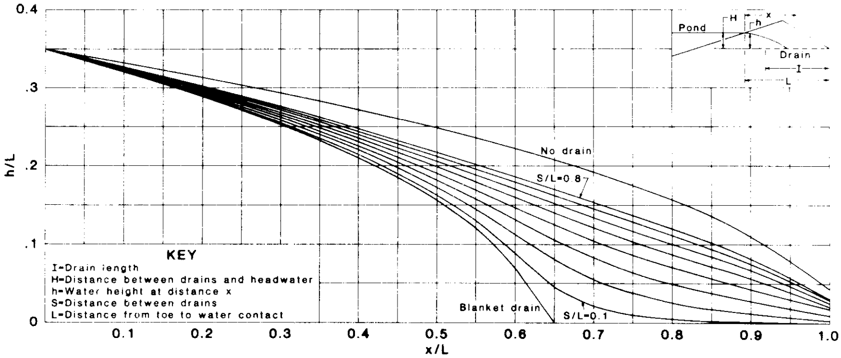

FIGURE 17. Dimensionless phreatic profiles, H/L = 0.35, I/L = 0.375.

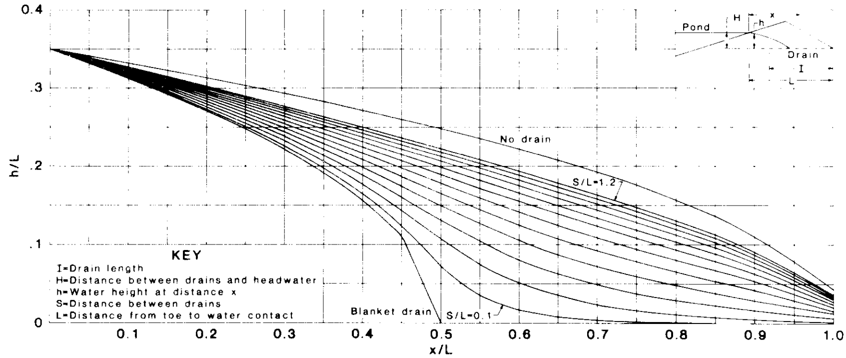

FIGURE 18. Dimensionless phreatic profiles, H/L = 0.35, I/L = 0.5.

I - drain length,
h - phreatic surface height,
x - distance from headwater to a point in the embankment cross section, and
S - distance between drains.

The curves, representing drain spacing (S/L), are bounded by a blanket drain curve and a "no-drain" curve. The criterion for increments of S/L was graph readability. The following example illustrates how the graphs are used.

1. Plot the cross section of the embankment. See figure 19 for the cross section used in this example.

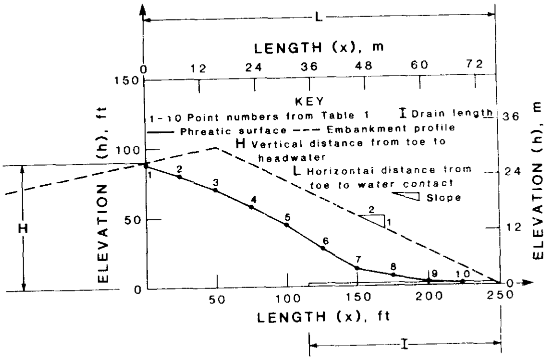

FIGURE 19. Embankment cross section plotted using dimensionless graphs.

2. Determine the distance between the drains and the headwater (H) and the horizontal distance from the toe of the embankment to the point at which the pond intersects the upstream slope (L). For this example, H = 27.43 m (90 ft) and L = 76.20 m (250 ft).

3. Calculate H/L. H/L = 27.43 m/76.20 m = 0.36.

4. Find the value of H/L from the graphs that is closest to the value calculated in step 3. For the example, the closest chart value is 0.35.[4]

5. Select the desired drain length ratio I/L. For illustration, 0.5 is chosen for I/L, representing a drain length of 0.5 x 76.20 m = 38.1 m (0.5 x 250 ft = 125 ft). The graph representing H/L = 0.35 (step 4) and I/L = 0.5 is figure 18.

6. Select the phreatic profile desired from the figure. The phreatic surface described by S/L = 0.2 is chosen for this example; this represents a horizontal drain spacing of 0.2 x 76.20 m = 15.24 m (0.2 x 250 = 50 ft).

[4]A more extensive set of charts is available in Bureau of Mines RI 8875 titled Estimating Horizontal Drain Design by the Finite-Element and Finite-Difference Methods.

7. From the S/L curve selected, read the values of h/L at several values of x/L. (x is the horizontal distance from the pond/embankment contact to a point on the horizontal axis, and h is the water height at point x.)

8. Calculate (x/L) x L and (h/L) x L. Table 1 shows conversions from the dimensionless values to units of the embankment used in the example.

9. Plot the products from step 8 on the cross section of the embankment.

10. Repeat the above process for various drain length and spacing combinations if desired.

Factor of safety analyses using the Simplified Bishop Method of Slices [2] were performed on the example embankment, without drains and with drains spaced 15.24 m (50 ft) apart (S/L = 0.2). Table 2 shows the conversion from the dimensionless curve values of figure 18 to units of the example embankment for the case of no drains.

The soil lying beneath the phreatic surface was assumed to be fully saturated and to have the following physical properties:

Angle of internal friction: 30°
Cohesion: 0 kPa
Density: 17,280 N/M^3 (110 pcf)

The soil above the phreatic surface was assumed to have no capillary zone and to have the following physical properties:

Angle of internal friction: 35°
Cohesion: 0
Density: 14,923 N/M^3 (95 pcf)

Figure 20 shows the phreatic surfaces for the two cases along with the respective slip circles. The example illustrates the importance of a low phreatic surface, since the factor of safety (FS) is increased from 0.51 for the case of no drains to 1.3 for the case of drains spaced 15.24 m (50 ft) apart.

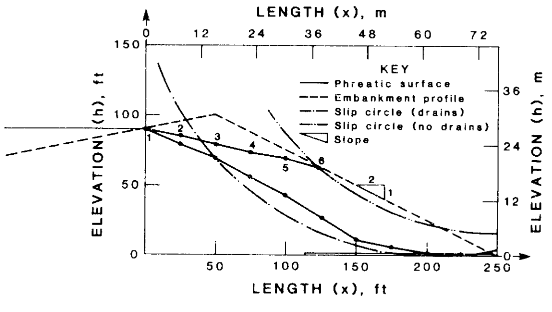

FIGURE 20. Slip circles for example embankment.

TABLE 1. - Conversion from dimensionless graph values to units used in example, drains 15.24 m (50 ft) apart

(H = 27.43 m (90 ft), L = 76.20 m (250 ft), S/L = 0.2, I/L = 0.5, and H/L = 0.35)

	Dimensionless graphs		Values calculated for example	
Point[1]	x/L	h/L	$\frac{x}{L} \bullet L$ m (ft)	$\frac{h}{L} \bullet L$ m (ft)
1	0.0	0.35	0.0 (0.0)	26.7 (87.5)
2	.1	.32	7.6 (25.0)	24.4 (80.0)
3	.2	.28	15.2 (50.0)	21.3 (70.0)
4	.3	.23	22.9 (75.0)	17.5 (57.5)
5	.4	.18	30.5 (100.0)	13.7 (45.0)
6	.5	.11	38.1 (125.0)	8.4 (27.5)
7	.6	.05	45.7 (150.0)	3.8 (12.5)
8	.7	.03	53.3 (175.0)	2.3 (7.5)
9	.8	.01	61.0 (200.0)	.8 (2.5)
10	.9	.001	68.6 (225.0)	.1 (.3)

[1]Points indentified in figure 19.

TABLE 2. - Conversion from dimensionless graph values to units used in example, no-drains case

(H = 27.43 m (90 ft), L = 76.20 m (250 ft), and H/L = 0.35)

	Dimensionless graphs		Values calculated for example	
Point[1]	x/L	h/L	$\frac{x}{L} \bullet L$ m (ft)	$\frac{h}{L} \bullet L$ m (ft)
1	0.0	0.35	0.0 (0.0)	26.7 (87.5)
2	.1	.34	7.6 (25.0)	25.9 (85.0)
3	.2	.32	15.2 (50.0)	24.4 (80.0)
4	.3	.29	22.9 (75.0)	22.1 (72.5)
5	.4	.28	30.5 (100.0)	21.3 (70.0)
6	.5	.25	38.1 (125.0)	19.1 (62.5)

[1]Points identified in figure 20.

CONCLUSIONS AND LIMITATIONS

Computer models of horizontal drains in tailings embankments are feasible design tools for predicting the height of the phreatic surface midway between drains. Results from two laboratory models and two field cases compare favorably to the computer models used in the study when applicable boundary conditions were used. More specifically -

1. The two-dimensional and three-dimensional finite-element codes calculate nearly the same results for the phreatic surface midway between horizontal drains.

2. The phreatic surface from the above codes lies between the phreatic surface from two laboratory models and slightly above the phreatic surface of one field application. The phreatic surface from the second field application matches the computer-generated phreatic surface closely. Difference between the computer-generated phreatic surface and the phreatic surface that was measured in the model and in the field can be attributed to some combination of the following:

   a. Potentiometric measurement error (clogged filters, measurement accuracy, etc.).

   b. The assumption in the computer models that permeability is not a function of location in the embankment.

   c. Variations in upstream pond elevations in the laboratory model and field applications.

3. Coupled with slope stability analysis, the dimensionless charts presented herein can provide an estimate of horizontal drain length and spacing dimensions necessary to achieve slope stability.

4. Drain diameter was not addressed and is an area recommended for further study.

5. Nonhomogeneous embankments may require futher analysis to determine drain placement [1].

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