CONSOLIDATION DRAINAGE OF FINE GRAINED MATERIALS

D. Armstrong
South Australian Dept. of Mines & Energy
191 Greenhill Road
Parkside
South Australia 5063

ABSTRACT

A method is described for the determination of hydraulic diffusivity which is equivalent to horizontal coefficient of consolidation, for fine grained materials in the field using vertical drainage wells.

The radial and vertical drainage effects on a clay due to a single drainage well are illustrated by a worked example.

CONSOLIDATION DRAINAGE OF FINE GRAINED MATERIALS

The engineering behaviour of fine grained materials (FGM) is highly dependant on moisture content and, when fully saturated, on pore water pressure. Under given load conditions, the removal of some of the pore water from a FGM will result in an increase in strength since more of the load is taken by solid particles (effective stress) as pore water pressure reduces.

This process is known as consolidation and may be achieved either by increasing the vertical stress by loading the surface above the area to be consolidated and providing drainage to remove the excess water, or by reducing the groundwater level by pumping from an adjacent aquifer which causes a differential in pore pressure between the FGM and the aquifer which promotes the movement of water out of the FGM without loading at the surface.

Brown [1] has demonstrated the importance of the coefficient of consolidation (C_v) in mine drainage. He also demonstrated the equivalence of C_v with hydraulic diffusivity (T/S). The coefficient of consolidation commonly used in civil engineering refers to drainage in a vertical sense whilst the transmissivity (T) employed in hydrogeology refers to the ability of a material to transmit water in a horizontal direction therefore Brown's statements are in need of qualification.

Strong horizontal lamination is a frequent feature of FGM. The vertical hydraulic properties, which are governed by the finest grained layers, may be at least an order of magnitude lower than the equivalent horizontal properties which are governed by the coarsest layers.

The concept of horizontal coefficient of consolidation (C_h) as discussed by Baron [2] is the true equivalent of the hydrogeological concept of hydraulic diffusivity in which the Transmissivity (T) is the product of Hydraulic Conductivity (K) and aquifer thickness (b), and the Storage Coefficient (S) is the product of Specific Storage (S_s) and thickness.

$$C_h = {}^T/_S = Kb/S_sb = {}^K/_{S_s} \quad .$$

By analogy

$$C_v = K'/S_s'$$

where K' = vertical hydraulic conductivity

S_s' = specific storage in a vertical sense

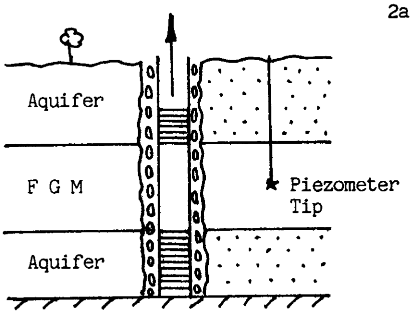
The evaluation of C_v is conventionally undertaken in the soils laboratory, however it is possible to determine K' and S_s' from discharge tests in the field following the procedure known as the Ratio Method described by Neuman and Witherspoon [3], or by a method proposed by Wolff [4].

The evaluation of the horizontal hydraulic properties of FGM in the field has not been a common practice due to the difficulties encountered in sustaining sufficient and measurable discharge from such materials in order that conventional analytical techniques may be applied to the field data.

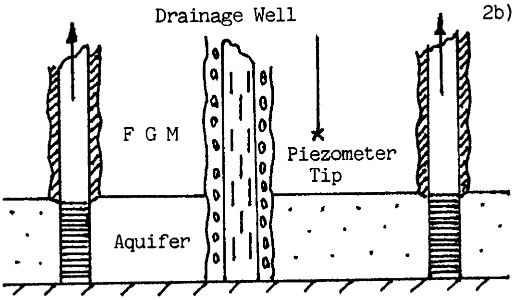
This paper proposes a method for determining C_h from field tests independant of discharge. It is based on the work of Glover [5] who presents an analytical technique for the evaluation of the flow from, and pressure distribution around, a flowing artesian well with constant drawdown.

The distribution of pressure around such a well is analogous to the distribution of pore water pressure around a drainage well in FGM with constant water level in the drainage well provided that the FGM remains saturated. The latter condition applies during the consolidation process.

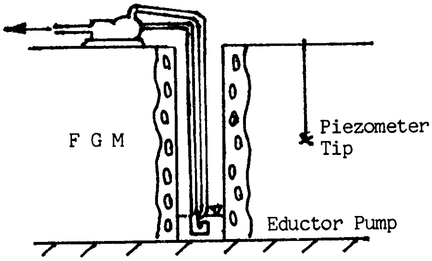
Glover produced a table (Table 3 ibid) relating relative drawdown (y/y_o), radial distance from the well (${}^r/_a$ where a = well radius) and $\sqrt{4\alpha t}/a$ where α = T/S, t = time.



2a) Combined Production/Drainage Well open to aquifer and F G M



2b) Single Drainage Well with multiple production wells to lower potentiometric surface in the aquifer



2c) Drainage Well (no aquifer) in FGM with Eductor Pump

FIGURE 2 Possible Layouts for Determination of C_h in the field

The table contains all the relevant information for the construction of a series of type-curves for analysis of flow from a flowing artesian well in unsteady state with constant drawdown in the absence of recharge or discharge boundaries.

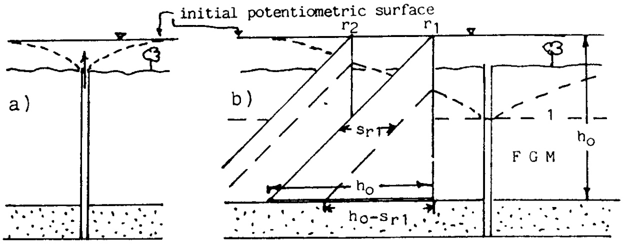


FIGURE 1 Analogy between flowing artesian well (a) and drainage well in F.G.M. (b)

1 = lowered potentiometric surface for aquifer due to discharge elsewhere

s_{r_1} = drawdown in aquifer at r_1 = available consolidation in F.G.M.

h_o = initial pressure head at base of F.G.M.

The same information may be applied to pore pressure reduction due to radial flow in F.G.M. around a drainage well (Figure 1) provided that the potentiometric surface can be lowered by either:

1. a single drainage well pumping from an adjacent aquifer with gravel pack opposite the F.G.M. to permit radial drainage to occur (Figure 2a)

2. a separate production well or wells pumping from the adjacent aquifer in which case the production wells must be cased and cemented opposite the F.G.M. interval in order to prevent radial drainage of the F.G.M. towards the production wells whilst the drainage well remains open to the F.G.M. (Figure 2b) or,

3. where no aquifer is present, a single drainage well open to the F.G.M., in which a constant head is maintained by means of an eductor type pump (Figure 2c).

In addition to production and drainage wells, at least one piezometer is required, preferrably of the pressure transducer or pneumatic porous tip type. Stand pipe piezometers may show a significant lag in response at early times due to the small volume of water moving in the F.G.M. in response to pressure changes and therefore are not recommended.

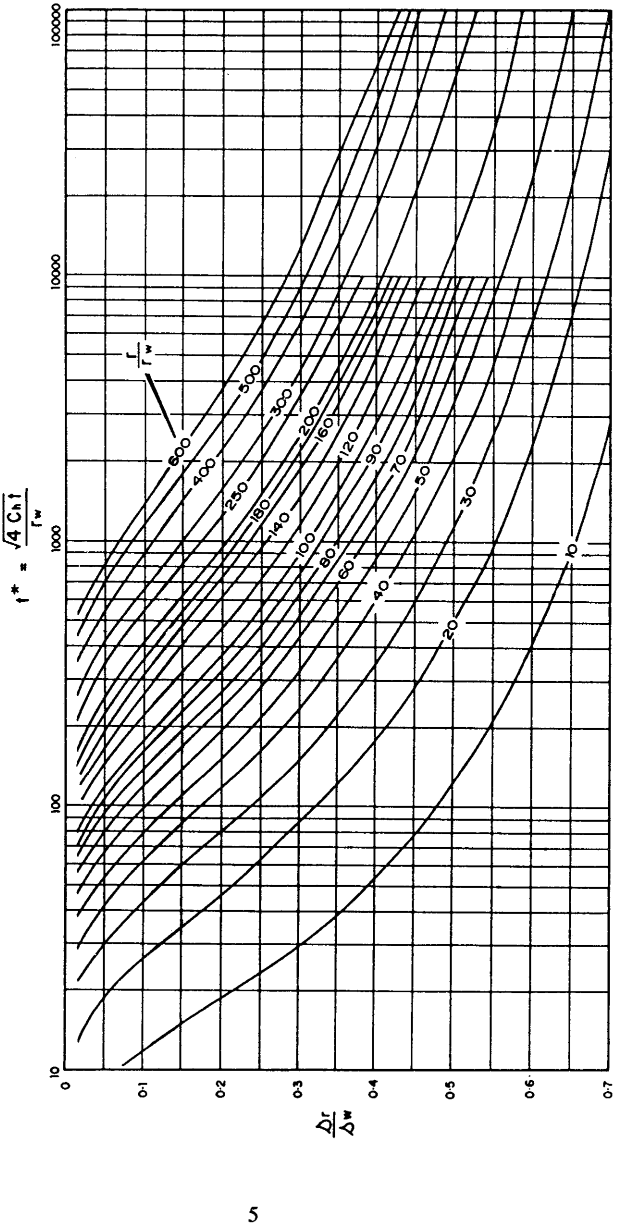


FIGURE 3: GLOVER Table 3 Type-curves

The consequence of delayed piezometer response would be a value of C_h which is lower than the true value.

The piezometer tip should be located at the half height of the F.G.M. if aquifers are present above and below, or if an aquifer exists on only one horizontal boundary, the tip must be located at the greatest possible vertical distance from the aquifer to avoid vertical consolidation effects at early time which are most marked close to an adjacent aquifer.

The piezometer should be located within a few metres of the drainage well to capitalise on the rapid response close to the well.

The duration of the field test should be of the order of days or weeks rather than the few hours to days normally employed for a conventional aquifer test. This is because of the relatively slow response of the F.G.M. due to low permeability.

Analysis of Data

A family of semi-logarithmic type-curves of y/y_0 (linear scale) versus $\sqrt{4\alpha t}/a=t^*$ (logarithmic scale) for a range of r/a values is plotted using data from Glover Table 3. Figure 3 is a set of type-curves in which Glover's parameter α is replaced by it's equivalent, C_h, and (a) is replaced by r_w.

$\sqrt{4C_h t}\,/r_w = t^*$ is known as the dimensionless time factor

Re-arranging the dimensionless time factor in terms of C_h gives:-

$$C_h = (t^*\ r_w)^2/4t \qquad (1)$$

The relative drawdown, which is the ratio of the observed pore water pressure reduction in the piezometer located at radius r (s_r), to the constant drawdown in the drainage well (s_w), is plotted (linear scale) versus $\sqrt{t}$ (logarithmic scale) on semi- log paper having the same modulus as the type curves on both linear scale (0-1.0) and logarithmic scale, to produce the field data plot.

The field data plot is then overlain on the type-curves and matched with the appropriate r/r_w type-curve by sliding left to right. The match point value is the t^* value corresponding to a field data plot value of $\sqrt{t} = 1$.

The match point values are $\sqrt{t} = 1; t^*$. The real time value of t = 1 and equation 1 becomes:-

$$C_h = (t^*\ r_w)^2/4 \qquad (2)$$

which enables C_h to be calculated by substitution of t* in equation (2).

Where t is measured in days and r_w in metres, C_h is in units of m^2d^{-1}, however since $T/S = C_h$ is usually small for F.G.M. it is often more convenient to operate in m^2yr^{-1}.

Deviation of the field data plot from the theoretical type-curve may provide useful information on the behaviour of the F.G.M. under radial drainage conditions.

Deviation below the type curve (ie. a steeper field data plot) indicates that pore water pressure reduction is more rapid than radial drainage theory alone can account for therefore vertical drainage may be significant.

Deviation below the type curve may also be due to:-

a) the presence of a coarser layer within the F.G.M. close to the piezometer tip

b) the presence of vertical defects (joints) in the F.G.M.

c) the removal of fines from the F.G.M. in the vicinity of the drainage well resulting in an increased permeability

Deviations above the type-curve indicate a pore water pressure reduction rate which is less than that predicted by radial drainage theory and which may be due to:-

i) radial arching in the F.G.M. with rapid consolidation and reduction in permeability near the drainage well

ii) exsolution of dissolved gases from pore water within the F.G.M. close to the drainage well producing bubbles which block pore spaces and reduce permeability

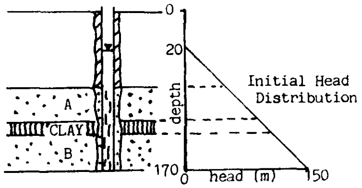
iii) blocking of pore spaces near the drainage well by physical movement of fines or chemical precipitation of solids in the low pressure zone causing a reduction in permeability

The use of C_h and C_v in the design of a multiple well drainage system for the relief of pore water pressure in F.G.M. will be the subject of a sequal to this paper.

It is appropriate however to discuss briefly the impact of a single drainage well in an infinite F.G.M. with the aid of the Glover Table 3 type-curves.

Consider a single drainage well which penetrates 2 sandstone aquifers separated by a 10 m thick clay (Figure 4).

The well is screened and gravel packed over the entire sandstone and clay interval and serves as a discharging well in addition to its drainage function. The initial head above the base of the lower sandstone is 150 m and under pumping conditions the drawdown in the well is 50 m.



(A) Sandstone $C_h=5x10^4$ m^2d^{-1} $(T=5m^2d^{-1}; S=10^{-4})$

(B) Sandstone $C_h=5x10^4$ m^2d^{-1}

Clay $C_h=2m^2d^{-1}$; $C_v=0.1m^2d^{-1}$

FIGURE 4 Single Drainage Well Example

Radial Drainage Effects

Assuming that the well rapidly attains its working drawdown of 50 m, then remains at this level, the type-curves of Glover Table 3 (Figure 3) may be used to analyse both drawdown in the aquifers and radial drainage effects in the clay.

We will consider drawdown and radial drainage effects at 5, 10 and 20 m radial distances from the well whose radius is 0.1 m. The drawdown in the aquifers above and below the clay will determine the maximum residual pore pressure in the clay and therefore the amount of available consolidation.

Given that r_w = 0.1 m and r = 5 m; 10 m; 20 m, we are concerned with type-curves for r/r_w = 50 ; 100 ; 200.

Drawdown in the aquifer will be examined after continuous pumping for 1, 5, 10 and 100 days by determining the t* value using equation (1) and reading the s_r/s_w value from the appropriate type-curve.

eg. $C_h = 5x10^4$ m^2d-1, $r/r_w = 50$, $r_w = 0.1$, t = 1 day

$$t^* = \sqrt{4\ C_h\ t}/r_w = 4472\ ;\ s_r/s_w = 0.52$$

$$s_w = 50\ m\ \text{therefore}\ s_r = 0.52x50 = 26\ m.$$

Table 1 lists values of s_r/s_w for each radius and time combination for sandstones A and B. A typical head distribution at time t is shown in Figure 5.

Time (t) days	r = 5m	r = 10m	r = 20m
1	.52	.45	.35
5	.56	.49	.41
10	.58	.50	.43
100	.64	.56	.49

TABLE 1 Relative Drawdown (Δ_r/Δ_w) Sandstone A and B

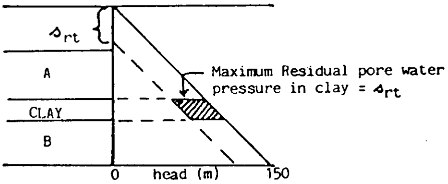


FIGURE 5 Head distribution in A and B at time t and radius r

Radial drainage of the clay commences as soon as pumping starts but progresses much more slowly than the development of drawdown in the aquifers.

The fraction of pore water pressure reduction or degree of consolidation due to radial drainage (Urt) is calculated in the same way as relative drawdown. The fraction of maximum residual pore water pressure remaining at time t due to radial drainage (u_{rt}) = 1 -Urt.

eg. $C_h = 2\ \text{md}^{-1}$; $r/r_w = 50$; $t = 100$ days.

$$t^* = \sqrt{4C_h t}\,/r_w = 283;\quad \Delta_r/\Delta_w = \text{Urt} = 0.29$$

Table 2 lists the degree of consolidation in the clay due to radial drainage (Urt) for each radius and time together with fraction of maximum residual pore water pressure (u_{rt}).

	Urt			u_{rt}		
Time (t) days	r = 5m	r = 10m	r=20m	r = 5m	r = 10m	r = 20m
1	0	0	0	1.0	1.0	1.0
5	.06	0	0	.94	1.0	1.0
10	.17	.05	0	.83	.95	1.0
100	.29	.16	.06	.71	.84	.94

TABLE 2 Urt and u_{rt} values for the clay

A comparison of table 1 and 2 shows clearly that the clay lags dramatically behind the aquifers in its response to radial drainage due to it's low C_h value compared to the aquifer.

Vertical Drainage Effects in the Clay

In response to the pressure difference between the clay and the aquifers, which is induced by drawdown in the latter, water moves vertically towards the aquifer in the process described by Terzaghi [6] as consolidation. The degree of vertical consolidation (Uv) varies with vertical position in the F.G.M. so for simplicity we shall initially consider only the average degree of vertical consolidation ($\bar{U}v$).

The rate of vertical consolidation is described by equation 3.

$$t_v = C_v t/H^2 \quad \text{(where H is the maximum flow path length)} \quad (3)$$

In the current example, H = half the thickness of the clay and if draining in only one direction (ie. the upper or lower boundary of the F.G.M. is impermeable), H = the full thickness of the F.G.M.

Figure 5 illustrates the relationship between the dimensionless time factor t_v, and average degree of consolidation ($\bar{U}v$) together with the average fraction of maximum residual pore water pressure remaining ($\bar{u}_v$).

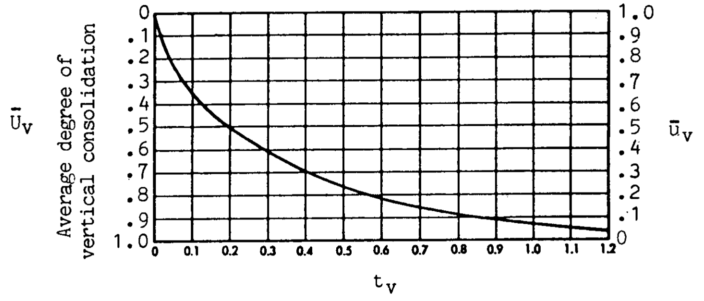


FIGURE 6 Average degree of Vertical Consolidation (modified after Terzaghi)

Equation 3 is used to calculate t_v values which are converted to consolidation values by reading from Figure 6.

The values of $\bar{U}v$ and $\bar{u}_v$ obtained are fractions of the available consolidation which is determined by the drawdown in the adjacent aquifers.

The average degree of consolidation, which is time dependant, not distance dependant, is calculated for each time step as follows:

$C_v = 0.1\ m^2d^{-1}$; $H = 10/2 = 5m$; $t = 10$ days

$tv = C_vt/H^2$

$= 0.04$; from figure 6, $\bar{U}v = 0.22$, $\bar{u}_{vt} = 0.78$

Table 3 shows, for the clay zone, $\bar{U}vt$ and $\bar{u}_{vt}$ for various times after pumping commenced.

time (t) days	$\bar{U}vt$	$\bar{u}_{vt}$
1	.07	.93
5	.17	.87
10	.22	.78
100	.70	.30

TABLE 3 Average Degree of Consolidation and Average Fraction of Maximum Residual Pore Pressure remaining at time t in the Clay

By comparison of tables 2 and 3 it can be clearly seen that for the example under consideration ($C_h/C_v = 20$), the effects of vertical drainage are dominant.

The combined effect of radial and vertical drainage is evaluated by equation 4.

$$u_{rvt} = \frac{u_{rt} \cdot \bar{u}_{vt}}{u_o} \qquad (4)$$

in which u_{rvt} = fraction of maximum residual pore pressure remaining due to both radial and vertical drainage at time t

u_o = 1 (ie. 100% maximum residual pore pressure)

Multiplying u_{rt} from Table 2 with u_{vt} from Table 3 results in Table 4 which shows the combined effects of radial and vertical drainage in the clay.

	$\bar{u}_{rvt}$		
time (t) days	r = 5m	r = 10m	r = 20m
1	.07	.07	.07
5	.22	.17	.17
10	.35	.26	.22
100	.79	.75	.72

TABLE 4 Fraction of Maxim Residual Pore Pressure Remaining after time t due to Combined Effects of Radial and Vertical Drainage in the Clay

Whilst radial drainage effects are theoretically constant with depth in a uniform F.G.M. at any given radial distance, vertical drainage is strongly dependant on flow path length hence vertical position relative to the aquifer or drainage layers.

If a more detailed distribution of consolidation is required within the F.G.M., average values for vertical consolidation can be replaced by values calculated for various heights above the base of the F.G.M. using equation 5 (modified after Smith [7]).

$$u_z = \sum_{m=0}^{m=\infty} \frac{2}{M} \left(\sin \frac{Mz}{H}\right) e^{-M^2 t_v} \qquad (5)$$

where u_z = fraction of maximum residual pore pressure remaining at height z above base of F.G.M.

M = $\pi(2m + 1)/2$

m positive integer from 0 to ∞

H = maximum flow path length

An alternative method for tv >0.05 is to read values directly from figure 7 which represents the two way drainage situation. The lower half of figure 7 may be used for one way drainage to an underlying drainage layer and the upper half for one way drainage upwards.

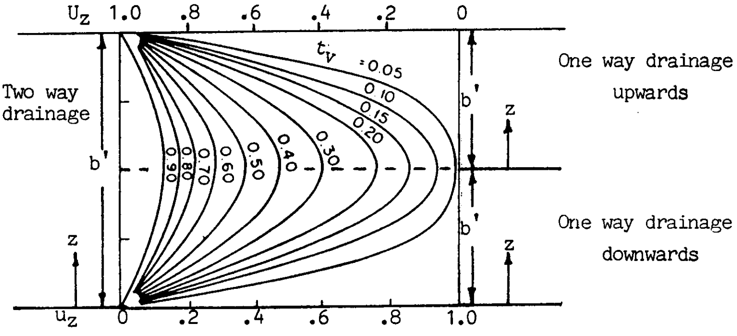


FIGURE 7 Vertical drainage response of an ideal F.G.M. to a step drawdown in head in adjacent aquifers (modified after Freeze and Cherry [8])

The lines of equal t_V in figure 7 are called isochrones and the average degree of vertical consolidation at time t, used in the earlier example represents the maximum residual pore pressure minus the area enclosed by the isochrone.

$$\bar{U}v = (2\ b'\ u_o - \text{area under ischrone})/2\ b'\ u_o \qquad (6)$$

Throughout this text fractional values for consolidation have been employed in order to allow for partial drainage which is a frequent occurrence in mining applications where it is often not economical to draw down the water level to below the base of the F.G.M.

To convert the fractional values to absolute pore water pressures in the case of partial drainage, such as in the given example, the residual pore water pressure remaining at time t can be determined from equation 7.

$$u_{rvt} = u_o - \left[(1-\bar{u}_{rvt})\ \Delta_r \right] \qquad (7)$$

with u_o and Δr expressed in kilopascals.

It is this residual pore pressure which, combined with the vertical loading, determines the effective stress and hence the engineering strength of the F.G.M. in the vicinity of mine excavations.

References

1. Brown, A. Third International Conference on Stability in Surface Mining. (Brawner C.O. Ed.) Am. Inst. Min. Metall and Pet. Eng. New York USA 1982)

2. Barron R.A. Consolidation of Fine grained Soils by Drain Wells. A.S.C.E. paper No. 2346. Trans. Am. Soc. Civil Eng. 113 (1948)

3. Neuman, S.P. and Witherspoon, P.A. Theory of flow in Aquicludes Adjacent to Slightly Leaky Aquifers. Water Res. Research 4.1 (1968)

4. Wolff, R.G. Field and Laboratory Determination of the Hydraulics Diffusivity of a Confining Bed. Water Res. Research 6.1 (1970)

5. Glover R.E. Transient Ground Water Hydraulics. Water Resources Publications. Fort Collins, Colorado. USA (1978)

6. Terzaghi, K. Erdbaumechanik auf boden physikalischer grundlage. Deuticke. Vienna (1925)

7. Smith, G.N. Elements of Soil Mechanics for Civil and Mining Engineers. Crosby Lockwood. London (1971)

8. Freeze, R.A. and Cherry J.A. Groundwater. Prentice Hall. New Jersey. USA (1979)