THE ELECTRICAL SEEPAGE ANALOGUE AND ITS APPLICATION IN TAILINGS DAM DESIGN

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ABSTRACT

In the analysis and design of tailings dams consideration must be given to the amount of water flowing through the structure and to the pore water pressures occurring within the soil.

This paper describes the application of a simple electric seepage analogue model which utilizes a resistance network of colloidal graphite lines in order to obtain two and three-dimensional flow nets for mine tailings dams. The theory of the model, its components and use are outlined. The model was found to be an expedient and economic design tool and can simulate simple and complex boundary conditions encountered in practical problems of mine tailings dams.

INTRODUCTION

There are many techniques available to provide exact and approximate solutions for seepage problems, both confined and unconfined. Examples of these are flow net sketching, exact and approximate analytical methods, finite difference and finite element methods, electrical analogue techniques using both resistance networks and conducting paper, and flow tank models. These and other methods have been described in most modern textbooks (e.g. Cedergren, [1]) and they all give results which provide useful information for the design engineer. The application of an exact analytical solution is rare in practice due to the fact of the complex geometry, the geology and anisotropic conditions encountered in real problems. The finite element technique has found wide-spread use in seepage problems (Desai and Christian, [2]) but it is dependent on the availability of suitable programs and skilled analysts. Also, experience with finite element programs has shown that some difficulties may be encountered in applying this technique to unconfined flow (Neuman and Witherspan, [3]). Flow tank problems are of limited use in particular for complex soil conditions with anisotropic flow properties. Flow net sketching has long been recognised as a powerful tool but is applicable only to isotropic and simple anisotropic conditions.

The rational design of mine tailings dams must take into consideration the seepage flow, both through the dam proper and its underlying foundation. This paper describes a simple and inexpensive electrical seepage analogue which has been used for a variety of practical problems, either for design or analysis purposes for mine tailing dams. The electrical analogue method is based on the theory that the flow of a fluid (water) through a porous medium (soil or rock) can be simulated by the flow of an electric current through a conducting medium. The porous medium is represented by a graphite resistance network. Meehan and Morgenstern [4] have given a brief introduction to this method and Tremblay [5] has shown its application to confined isotropic and anisotropic flow conditions of water into cofferdams. In this paper this method is applied to two-dimensional confined and unconfined flows of water through and under dams as well as to three-dimensional flow around abutments.

THEORY OF MODEL

The flow of current in a conducting medium as a result of applying an electrical potential is equal to the flow of water (incompressible fluid) through a porous medium under a hydraulic head. In the use of the electrical analogue model the porous medium is represented by a graphite resistance network and the permeability, water flow and pressure head are analogous to the conductance, current and voltage drop respectively of the model.

Darcy's law of water flow through an element of soil is given by

$$v = k\,i = k\,\frac{h}{l} \qquad [1]$$

where h = loss of head across a length l of the element
v = discharge velocity, and
k = coefficient of permeability or hydraulic conductivity

Written in a directional form along the x-direction, equation [1] becomes

$$v_x = k_x\,\frac{h_x}{l_x}$$

or the quality of flow across a cross section A_x is then

$$q_x = A_x k_x\,\frac{h_x}{l_x} \qquad [2]$$

The electrical analogue is based on the equivalence of flow q_x to current I_x, and total head h_x to potential drop V_x, or

$$I_x = \frac{V_x}{R_x} \quad \text{(Ohm's Law)} \qquad [3]$$

where I_x = current passing through resistance R_x, and
V_x = potential drop across R_x.

Now from the electrical analogy

$$V_x = mh_x \quad \text{and} \quad R_x = n \frac{l_x}{k_x A_x}$$

Therefore $I_x = \frac{m}{n} q_x$ [4]

where m and n are constants.

Then

$$\frac{q_x}{(k_x\ h_x)} = \frac{I_x\ (\frac{n}{m})}{k_x\ (\frac{V_x}{m})} = \frac{n}{(k_x\ R_x)} \qquad [5]$$

substituting $R_x = n \frac{l_x}{k_x A_x}$ into equation [5] ,

one obtains

$$\frac{q_x}{(k_x\ h_x)} = \frac{A_x\ (\frac{R_x}{l_x})}{R_x} \qquad [6]$$

let $r_x = \frac{R_x}{l_x}$ be the unit resistance along a line in the x-direction, then equation [6] will become

$$\frac{q_x}{(k_x\ h_x)} = \frac{A_x\ r_x}{R_x} \qquad [7]$$

The above expression gives then the quantity of flow through an element of soil in the x-direction and is proportional to the cross-sectional area, its linear unit resistance and inversely proportional to its total resistance over a length l_x.

Similarly the total flow between some set boundaries of a physical problem can be derived in a like fashion,

$$\frac{Q}{k_x\ H} = \frac{A_x\ r_x}{R} \qquad [8]$$

where H = total head loss, and
R = total resistance of the model measured across the extreme boundaries.

It is also obvious from equation [5] that

$$n = A_x\ r_x\ k_x \qquad [9]$$

The above relationship must be satisfied in all directions and regions of the model, that is,

$$A_x\ r_x\ k_x = A_y\ r_y\ k_y = A_z\ r_z\ k_z = n \qquad [10]$$

This is of vital importance in order that appropriate grid dimensions can be drawn according to the corresponding permeabilities. Throughout this investigation square or rectangular grids have been used mainly for convenience sake. Other researchers (i.e. Butterfield and Howey [6]) have used triangular grids with similar results. A triangular mesh system is more time consuming as compared to a square or rectangular network.

COMPONENTS OF MODEL

General

The components of the electrical analogue model is best illustrated with reference to a two-dimensional flow problem through a dike as shown in Figure 1. Figure 1(a) shows seepage occurring through and under a tailings dam. To construct the graphite network as shown in Figure 1(b), the geometry of the problem and the boundary conditions are first sketched on a good quality drafting paper. A square grid representing an isototropic porous medium is then drawn with an adjustable ink pen using a colloidal suspension of graphite. Once the ink has dried, electrical potentials can be applied across the known boundaries (across lines AB and CD in Figure 1(b)). The potential along line AB is 100 and along line CD is 0 percent respectively. The potential of the probe, placed at any nodal or line point within the grid, can be read with a galvanometer. Any number of desired equipotential drops can thus be determined.

For a homogenous tailings dam where the horizontal permeability is twice that of the vertical, this can be modelled by doubling the spacing of the vertical grid lines thereby creating rectangular grid elements having a length to width ratio of two, equal to the ratio of the permeabilities. In the second method, the square grid is retained but the resistance of the vertical lines is double of the horizontal lines. The resistance can be varied by adjusting the width of the drawing pen. The second method is preferable where the permeability ratios are greater than five. (The two techniques are illustrated in Figure 2). Impervious boundaries, such as clay or asphalt cores, or upstream clay blankets, are easily modelled by breaking the grid. The boundaries of the model, such as extreme flow and equipotential lines to which the potentials are applied, are drawn with a highly conductive silver paint. Sunshine Scientific Instrument Silver Paint No.C24 and General Cement Manufacturing Company Paint No.21-1 have been found very satisfactory for this purpose.

The difficulty in modelling a particular permeability profile is controlling the resistance of the grid lines. The resistance in ohms per unit length of the line is measured with an ohm-meter to obtain the average linear resistance of the various lines in the grid. For a given line width slight variations in line resistance are unavoidable even with a good drawing technique. For example, a series of lines with an average resistance of 1,000 ohms per mm will have variations from 800 to 1,2000 ohms/mm. The error will be distributed at random over the entire grid. The errors appear to have little effect on the results (Meehan and Morgenstern [4]). In order to further minimize variations in line resistance, the lines should be drawn in alternate directions as shown in Figure 2(a).

Conducting Fluid

The graphite solution used for the experiment was the Electrodag 154 graphite which is marketed as a fluid of 20% concentration. The combination of electrical properties and easy flow is highly valued in drawing graphite resistance networks. A wide range of electrical resistances could be obtained by varying the concentration of graphite in the ink solution. Isopropanol to graphite concentration of 0.25:1 to 0:50:1 were found very satisfactory for the problems on hand.

Apparatus

In most applications of electrical analogue systems, a voltage-sensing device is employed to detect points of equal potential throughout the field. The sensing of voltage is preferred in analogue systems because it can be achieved by means of a probe, while the sensing of flux or current presents a more complex problem. The voltage-sensing device should have the following properties: (Karplus, [7])

1. It must be capable of making positive contact with the conductive sheet or ink.
2. It must not have a sharp point or the conductive medium will be damaged.
3. It must be sufficiently hard so that it is not worn down and flattened by repeated traverses over the conductive medium.

A probe with a dull tip was found to be most suitable for this purpose. A so-called field plotter (type F P 144, manufactured by Servomex Control Limited, U.K.) was used throughout this investigation.

Sources of Error

According to Karplus [7] inaccuracies in the solution of Laplace's equation by means of conductive mediums originate from three basic sources which are:

1. The conductive medium does not simulate correctly the properties of the soil in the field.
2. The boundary conditions of the actual soil strata are not simulated correctly in the analogue model.
3. Errors occur in the sensing and plotting of the equipotential lines (or flow lines) of the analogue system.

Sources of errors mentioned in points 1 and 2 arise mainly from inadequate field investigations of the soil profile and have little to do which the technique in the solution of the problem. It is well accepted that in any solution of water flowing through a real soil simplifying assumptions with regard to stratigraphy have to be made. With respect to point 3 one can guard against this source of error by using extreme care in drawing the grid and in taking the network measurements. It has been found that placing a hand on top of the grid while taking readings with the probe will affect the results. Any objects, such as pencils, drafting equipment should also not be placed on the network.

APPLICATION OF MODEL TO TAILINGS DAMS

Several practical problems of water flowing through and under mine tailings dams have been investigated. Generally the phreatic line, or top flow line, is not known and one has to employ approximate methods such as the well known Casagrande graphical solution or Dupuits equation.

The electrical graphite analogue can be used in successive approximations to find the correct phreatic line as was described by Uginchuss [8]. This method is illustrated with reference to Figure 3 which shows the model of a homogeneous tailings dam. The total head between the upstream and downstream water levels is divided into a convenient number of equal potential drops, say 10, as shown in Figure 3. The electric potential of V volts is applied at the upstream potential line AB and the variable potentiometer, which is set at the 90% potential, is connected at the point of intersection of the downstream slope of the dam and the 0.9H horizontal line. The 90% potential drop has to lie along this line. The exact point is located with the movable proble. Similarly the remaining points are located with the probe with the galvanometer set at the proper potential. Connecting all these points will yield the curve AC_1. This curve is a first approximation. The model is now cut just above this line, and the procedure is repeated to obtain a second approximation of the phreatic line AC_2 and so on until the position of the top flow line remains virtually unchanged. This method is equally applicable for tailings dams which have either vertical, sloped, underdrain or toe filters.

ANISOTROPIC SOIL CONDITIONS

Homogeneous anisotropic flow conditions through dams can easily be handled by the model either by changing the spacing of the grid lines or by varying the resistance of the lines according to the ratio of the horizontal to vertical permeabilities. Figure 4(a) shows the flow net of a tailings dam has a horizontal permeability four times greater than the vertical one. The dam has an underdrain filter at the downstream slope of the dam. Figure 4(b) shows a similar dam with a sloping filter and a permeability ratio of 1 to 25. The model can also be applied to cases where the dam is made up of layers, each layer having different permeabilities in the vertical and horizontal directions (Figure 4(c)). It would be almost impossible to handle this situation with any other method. The tailings dam shown in Figure 4(d) is constructed from two different materials having a permeability ratio of 1 to 5. The flow net of material 1 consists of rectangles with side ratios of 1 to 5 and the flow net for material

2 are squares. Garga and Vargas [9] have applied the electronical analogue model also to cases of transient flow, such as rapid draw down in dams, with great success.

THREE-DIMENSIONAL FLOW

The model was also applied to three-dimensional flow which occurs with water seeping around the two abutments of a tailings dam. In such a case a convenient number of vertical sections parallel to the upstream-downstream direction have to be made through the abutment. All sections have to be studied simultaneously. This procedure becomes a bit intricate and needs some experience by the technician.

CONCLUSION

This paper has given a brief summary of the electro-analytical method as applied to two- and three-dimensional flow problems occurring in tailings dams. Extreme care and a certain skill are required to obtain good results. The method presented is of particular value in the design stage of a tailings dam when several alternate proposals are considered and each case can be readily analysed to obtain an economic and safe final design.

REFERENCES

1. Cedergren, H. R. Seepage, Drainage and Flow Nets, McGraw-Hill, New York, 1967.

2. Desai, C. S. and J. T. Christian. Numerical Methods in Geotechnical Engineering. McGraw-Hill, New York, Ch.14, 1977.

3. Neuman, S. P. and P. A. Witherspoon. Finite Element Method by Analysing Steady Seepage with a Free Surface. Water Resource Research, V.6, N.3 p.889-897. 1970.

4. Meehan, R. L. and N. R. Morgenstern. The Approximate Solution of Seepage Problems by a Simple Electrical Analogue Method. Civil Engineering, V.63, N.738, p.65-70. 1968.

5. Tremblay, G. A. Seepage Determination by the Electrical Analogue Method. Internal Report. Civil Engineering, Carleton University, Ottawa, 1980.

6. Butterfield, R. and I. R. Howey. A Simple Analogue for the Rapid Solution of General Flow Problems in Two-Dimensions. V.23, N.1, London. 1973.

7. Karplus, W. J. Analog Simulation, McGraw-Hill, New York, 1958.

8. Uginchuss, A. A. Seepage Through Earth Dams, Israel Program for Scientific Translations, Jerusalem, 1966.

9. Garga, V. K. and E. A. Vargas, An Electrical Seepage Analog and its Use in Earth Dam Design, paper submitted to ASCE - Geotechnical Division.

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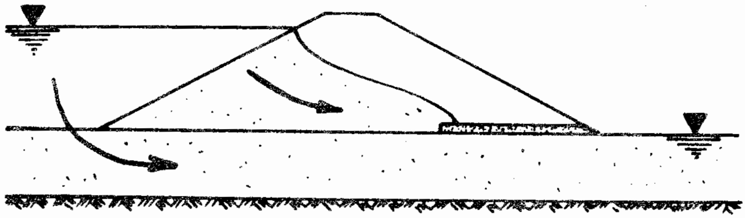

FIG. 1(a). Flow of Water Through Tailings Dam

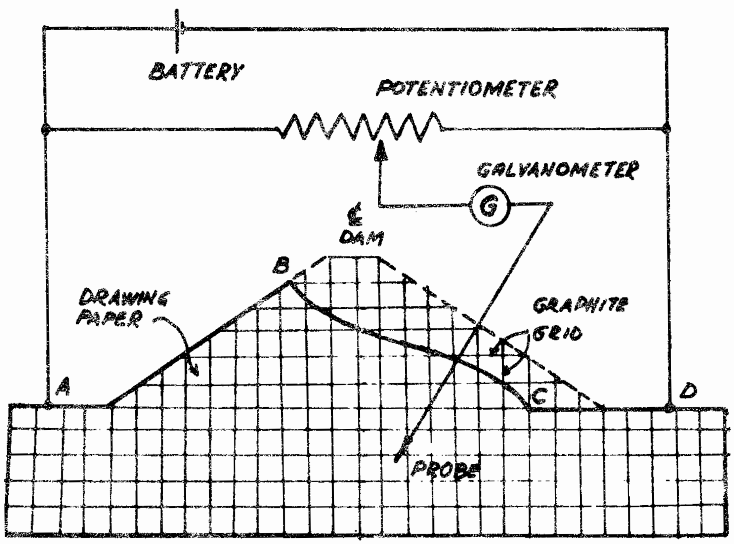

FIG. 1(b). Components of the Electrical Graphite Model

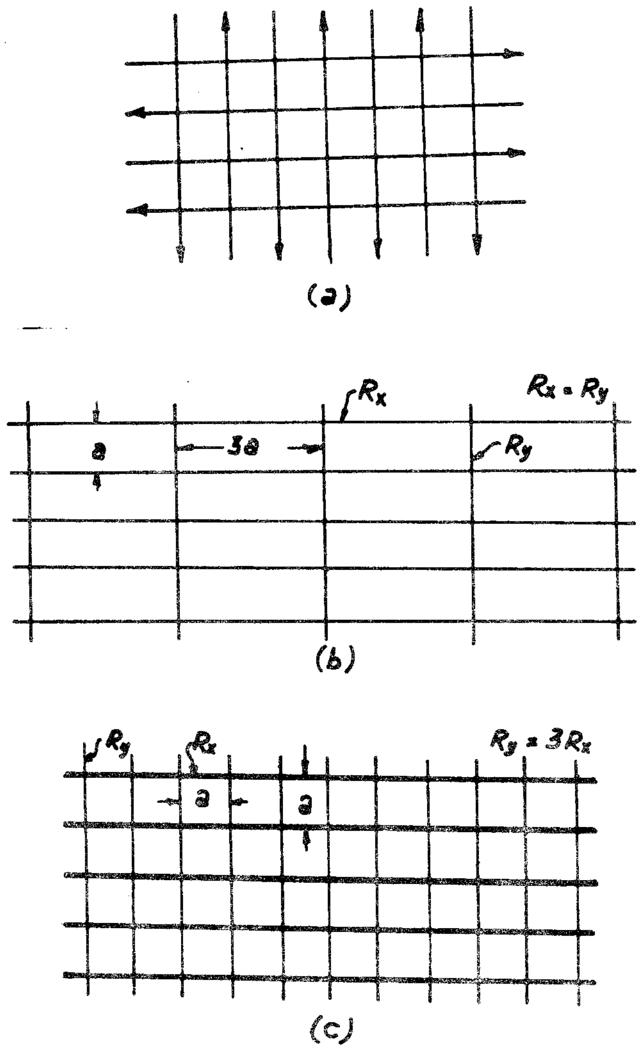

FIG. 2. Two Methods of Modelling Anisotropic Soils

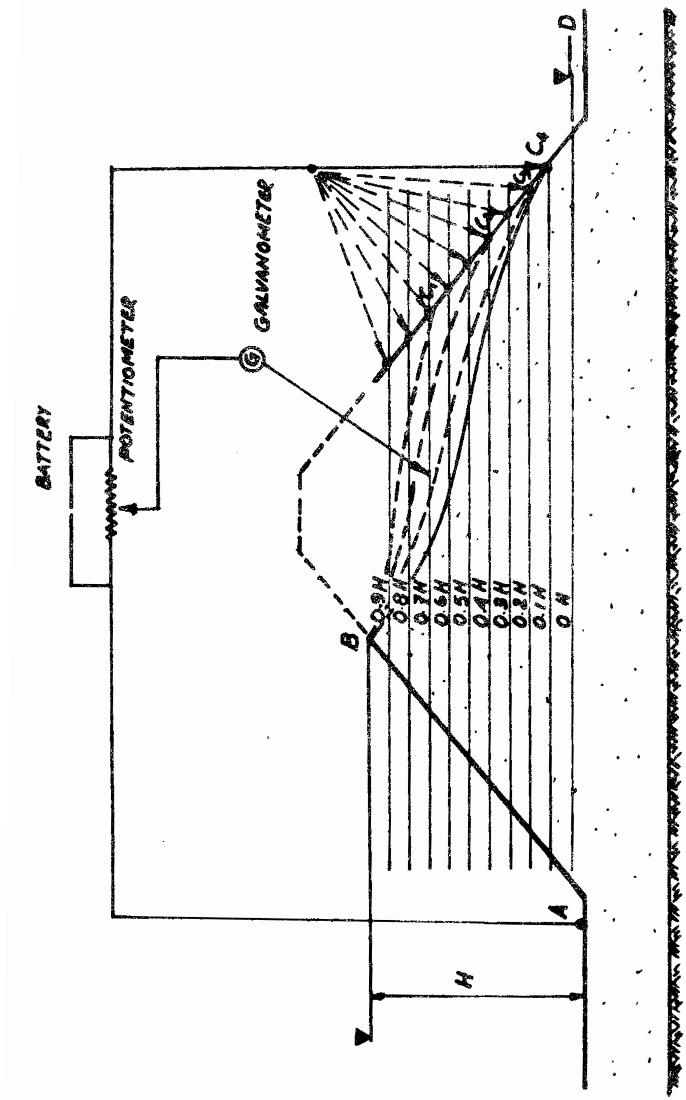

FIG. 3. Determining the Top Flow Line (Phreatic Surface)

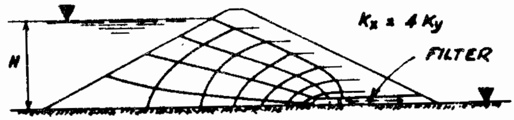

FIG. 4(a). Anistropic Flow Through Dam (Underdrain Filter)

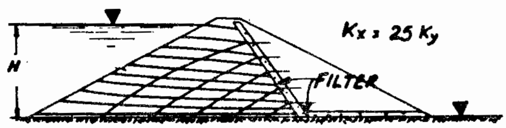

FIG. 4(b). Anisotropic Flow Through Dam (Chimney Filter)

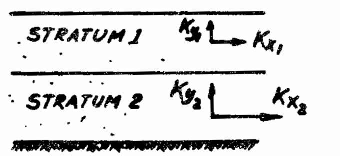

FIG. 4(c). Layered Anistropic Soil Conditions

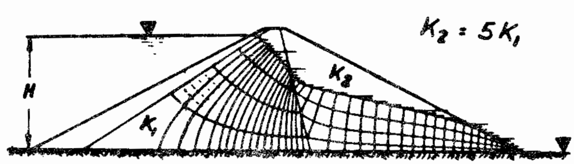

FIG. 4(d). Flow Through Dam Constructed of Two Materials