A BIFURCATION THEORY MODEL OF MINEWATER INRUSHES

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ABSTRACT

Bifurcation theory is applied to study the phenomenon of water inrush from an aquifer into an underground opening. To reach the opening, water must travel through a soft impermeable layer called "protective layer", such as clays, marls. Inrushes occur when hydrofracturing of this protective layer takes place.

Earlier studies were based on an iterative calculation of one phase models of the rock water interaction. Such models can essentially be used for forecasting but hardly for inrush control purposes. In this investigation, bifurcation theory, a branch of modern mathematics rooted in topology, is used. This approach can model processes characterized by sudden jumps such as sudden transition between microseepage and water inrush. A specific aspect of bifurcation theory, called catastrophe theory, is used to model the inrush phenomenon. The study includes:

- a brief review of catastrophe models and applications
- a presentation of analytical model of the water inrush phenomenon
- approaches of model fitting to empirical data
- some practical byproducts of the investigations.

First, the surface supporting the state trajectory is depicted; similarities between this surface and a cusp catastrophe surface are pointed out. Then, the potential function of the cusp catastrophe surface is given as a bimodal density function of mine water inrush yield. A statistical fitting of this stochastic type of catastrophe model to empirical data is then undertaken: procedures for

model fitting and data processing are given. Results of fitting the present model to empirical data are provided. Finally, some byproducts for practical applications, in particular, qualitative considerations leading to actual control decisions, are discussed.

INTRODUCTION

The phenomenon considered in this study is mine water inrush from an aquifer through an impermeable layer into a mining opening. Mining experience, laboratory findings and analysis of the process of water inrush show that the phenomenon begins as a water microseepage. After a strong and sudden change of the water conductivity of the impermeable layer, a water inrush starts. The possibility of modeling the mine water inrush using catastrophe theory is investigaged. The paper presents:

. A description of the mine water inrush phenomenon and a background on mathematical modeling efforts to date

. the limitations of the earlier model and the necessitiy for improvements

. the main questions which we would like the model to answer

. a brief discussion of bifurcation theory with special consideration of a stochastic approach to the cusp catastrophe model

. approaches for a new inrush model using catastrophe theory

. the procedures for empirical data processing and trials for model fitting

. recommendations concerning the possibility of using bifurcation theory and suggestions for further investigation of water inrushes

. some practical byproducts of the theoretical investigations on process control.

WATER INRUSH MODELING

Terminology

The terminology on mine water inrush process is illustrated in Fig. 1.

The protective layers are soft clays, marls and even hard rocks. In the protective layers there often are tectonic faults, and/or fissures caused by mining operations.

The aquifer may correspond to a saturated sandy soil, karstic or other fissured rock, an abandoned and flooded mine; water may also come from any surface reservoir /lake, river, sea/.

Existing Models

Water inrush is provoked by a mechanical interaction between the rock and the water systems [5].

The general differential equations of energy and impulse transport and mass balance of the rock-water system cannot be solved in the case of mine water inrushes because they contain empirical parameters and functions which are, for the most part, unknown. Therefore, the process can only be modeled step by step using successive one-phase approximations of the two-phase process /water and rock movement/ [8].

The steps for reaching a solution are as follows:

a. Describe and model the onset of seepage in the protective layer.

b. Model the widening process of the fissure caused by the effect of water. /a/ and /b/ are the so-called partial models.

c. Develop an approximate model of the complete water inrush process using the partial models [8].

a. Modeling the Onset of Water Inrush

In this step, the two-phase process of rock and water movement is approached as the one-phase process of water movement. The results of extensive investigations of empirical data suggest that the start of the inrushes can be determined by a threshold value of the hydraulic gradient I_o^*.

$$I_o^* = \frac{s}{m} \qquad /1/$$

This value is called the threshold gradient of the protective layer, because it is not equivalent to the threshold hydraulic gradient of seepage I_o. Investigations [8] show that $I_o < I_o^*$; consequently, micro-seepage and/or small inflows may occur, where inrushes do not.

The threshold gradient of the protective layer (I_o^*) have to be expressed as a random variable conditioned on rock stress, exploration borehole network and thickness of the protective layer. The most important factor is the rock stress [7].

Fig. 2 shows the empirical probability density functions (pdf) $p(j)$ and distribution functions (DF) $F(j)$ for different protective layers in Hungarian mines [7]. These probability functions give us the expected number N of water inrushes in a given mining operation as:

$$N = N_A \cdot F(I_o^*) \qquad /2/$$

or for a given value of the thickness of the protective layer (m) a threshold value of water head (s_o^*) and its distribution function $F(S_o)$ can also be used:

$$N = N_A \cdot F(s_o^*) \qquad /3/$$

where N_A is the number of the water inrushes without protective layer.

Laboratory experiments on hydrofracturing of protective layers and field experiments on reservoir-hydrofracturing in petroleum industry [20] show that the relationship between the mean value of the hydraulic threshold gradient I and the mean value of the minimal normal rock stress σ_m can be approximated by a linear function with proportionality constant c for a given interval of o.

$$\sigma = cI_o^* \qquad /4/$$

and using Eq. /1/

$$\sigma = c\frac{s_o^*}{m} \qquad /5/$$

For a given thickness m of the protective layer, letting

$$c' = \frac{c}{m} :$$

$$\sigma = c' \, s_o^* \qquad /6/$$

where s_o^* is the threshold value of the water pressure for a given thickness (m) of the protective layer.

b. Fissure Widening Process /Piping Process/

Approximate analytical models of rock mechanics have been used in which the force exerted by water was taken as a constant. Salient results of these approximate analyses are as follows.

The most important factor in the widening of fissures during the first phase of widening is the static water pressure [7].

If the static water pressure $p < \sigma_{min}$ /where σ_{min} is the smallest principal component of rock pressure/, the fissures cannot be opened, and water inrush does not occur /but microseepage and small inflows may be present/. If $p > \sigma_{min}$, the fissures may be subject to a widening which will be a function of p, o, and the rock properties. In soft clays, static water pressure can open hairline tectonic fissures to a width of several millimeters which is sufficient for the beginning of intensive water flow. During the intensive water flow, also called the second phase of widening, Bobok and Somosvari [15] have shown, that the dynamic action of pure water may be neglected whenever chemical effect /defloculation/ [14] cannot occur. Therefore the most important dynamic effect is usually the erosion effect of solid particles [8] in the water flow.

In those cases where the aquifer is in loose sand, the clays have practically no hydraulic resistance. But in cases where the aquifer is in solid rock such as a karstic limestone, the clays with the same material properties have a hydraulic resistance which is overcome only by the widening of fissures; then only the loose granular zone in the tectonic fissures of the protective layer can be eroded by water.

Approximate Model of the Mine Water Inrush Process

Two cases will be considered

/i/ Once water inrush begins, the resistance R_p of the protective layer is eliminated.

/ii/ The protective layer continues to offer a known hydraulic resistance after the start of water inrush.

According to the analyzed field data and laboratory tests [7] inrush occurs whenever

$$p = \Delta s \cdot \gamma > \sigma_{min} \qquad /7/$$

in which is the specific gravity of water.

As a first approximation, seepage is taken as time invariant and two sets of static hydraulic parameter values are considered: the first one describes pre-inrush conditions, and the second, conditions following inrush commencement.

c. A Simplified One-Phase Model of the Phenomenon

For given stress conditions, protective layer thickness and stationary seepage flow the model is given as [17]

$$s - s_o = \alpha q \quad \text{when} \quad s < s_o^* \quad \text{/microseepage/} \qquad /8a/$$

$$s = \alpha' q + \beta' q^2 \quad \text{when} \quad s > s_o^* \quad \text{/water inrush/} \qquad /8b/$$

where α and β are the hydraulic parameters before water inrush, and α' and β' are the parameters during and after the water inrush. α and β are determined by the initial parameters of the protective layer because the hydraulic resistance of the aquifer R_a is much smaller than the resistance of the protective layer R_{pl}.

$$R_a << R_{pl} \qquad /9/$$

Therefore, prior to an inrush occurence, the initial process is a microseepage through the protective layer.

In the case when

$$R_{pl} \cong 0,$$

α' and β' are the parameters of the aquifer.

As Fig. 2 shows, s_o^* is given by a distribution function $F(s_o^*)$ in a given interval.

$$s^*_{o\,min} \leq s_o^* \leq s^*_{o\,max} \qquad /10/$$

Consequently, "inrush" and "non-inrush" cases are simultaneously occurring in the interval defined by all Eq. /10/.

The number of inrushes is given by Eq. /2/ or Eq. /3/ and the number of non-inrush cases by:

$$N_O = N_A \left(1 - F(s_o^*)\right) \qquad /11/$$

Mining experience shows that the state variable is subject to a relatively sudden change of jump governed by the aforementioned fissure widening process.

Because of the inhomogenity of rocks both in the reservoirs and protective layers, α , α', and β and β' are random variables. The randomness of the microseepage $P_o(q)$ cannot be observed directly. The distribution of inrushes for given values of s and m are given by empirical data $P'(q)$. Theoretical analyses and empirical data show that the distribution of inrush yield and the water conductivity can be approximated by lognormal probability functions [12, 13].

A Simplified Two-Phase Model

a. State Variable

The state variable is the yield of the mine water inrush, because it is the only variable which can be directly measured.

b. Control Variables

The two most important control variables /for a given thickness m of the protective layer/ are, respectively, the water pressure characterized by head s and the rock stress characterized by the mean value of the minimal vertical rock stress σ_{min}.

c. State Transition

The state transition function is, for a given thickness m of the protective layer, Eq. /5/, that is:

$$\sigma_{min} = c' s_o^*$$

together with Eqs. /8a/ and /8b/. A qualitative representation of the state transition surface can be seen in Fig. 4 in the space (q, σ_{min}, s).

d. Density Function of the State Variable

As discussed in Section "Approximate Model of the Mine Water Inrush Process" the state surface represents only average values of the state variable. Because of the inhomogenities of rock parameters, the state variable must also be characterized by a probability density function of all possible states, as follows:

$$P(q) = P_o(q) \cdot (F(s_o^*) + P'(q) \cdot (1 - F(s_o^*)) \qquad /12/$$

where $P_o(q)$, $P'(q)$, $F(s_o^*)$ are functions of 9 and σ /for a given constant value of m/

If $s < s_{o_{min}}$

$$P(q) = P_o(q)$$

If $s > s_{o_{max}}$

$$P(q) = P'(q)$$

P(q) is a locally bimodal density function.

The sizes of micropores and the large fissures are all characterized by lognormal density functions [17], consequently the density functions of the yield of microseepage and of the inrushes are also characterized by lognormal density functions [12, 13].

According to the above considerations, the bimodal density function of the state variable are illustrated in Fig. 6. Two variants of the state variable are presented:

case a. state variable is q
case b. state variable is lg q.

MODEL GOALS

A new mine water control method is being developed in Hungary, whose essence is to deal preventively with rock-water /mechanical/ interaction in the protective layer [5, 6]. For studying this new method and for managing other mine control schemes, a two-phase model is needed. The one-phase approach allows for only one-directional change of the state variable.

It is recognized that only better developed physically based models will give really adequate tools for process control. In the meantime, an approximate mathematical model which can collapse two control variables into one presents a sudden jump may be of any help.

For a mathematical model of sudden jumps, catastrophe theory seems to be well suited even in the presence of random parameters [11]. Let us present a brief introduction to catastrophe theory with emphasis on the cusp catastrophe model which possesses one state variable.

APPLIED CATASTROPHE THEORY

The mathematical basis of catastrophe theory was originally developed by Thom [16] as a means of classifying singularities of differentiable functions. For most differentiable functions involving three variables, Thom's theorem states that the set of critical points is composed of folds, cusps, and ordinary points; similar statements can be made for functions of from one to six variables. The theorem itself is of little help for modeling purposes, as Sussman and Zahler [19] point out, since the statement is very general and does not predict the exact behaviour of the process being modeled.

In other work, however, Thom [16] and Zeeman [18] have proposed that many natural processes involving one state variable and two control variables display dynamic behaviour which can be modeled with the canonical potential function

$$f = 1/4x^4 + 1/2ax^2 + bx \qquad /13/$$

in which a and b are control variables and x is a state variable. Thom and Zeeman then assert that these processes tend to evolve toward states in which Eq. /13/ is minimized.

Minimization implies that, at equilibrium, for fixed a and b, values of x will tend to that set of points given by:

$$\frac{df}{dx} = 0 = x^3 + ax + b \qquad /14/$$

This solution set forms the cusp manifold in three dimensional space /see Fig. 5/.

Zeeman [19] has defined four qualitative characteristics which, if displayed by a system, suggest that a model based on catastrophe theory might be appropriate:

/1/ The behaviour of the process is bimodal over part of the system's range, with "sudden" changes occuring between levels of the state variable.

/2/ Transition between levels of the state variable occurs at different levels of the control variable, depending on the direction in which the control variables are moving /hysteresis/. Values of the control variables at which these transitions occur form the bifurcation set in a-b space.

/3/ Intermediate values of the state variable between the two extremes do not occur or are improbable. This corresponds to the middle sheet on the cusp, i.e., a region in which the critical points are unstable maxima.

/4/ Finally, if the process is such that the control variable a is not constant over the time period in which the control variable b and the state variable x change, for small perturbations in the initial value of x, large changes in the behaviour of the system /called divergence/ are possible.

All the above features are sketched in Fig. 5.

For multimodal distributions, Cobb [2] has developed stochastic catastrophe models which may be of use in our study.

Examples of Application

There are three principal groups of applications of catastrophe models. The first group of examples contain qualitative catastrophe models for describing economic, psychological and political processes.

In such qualitative models, the two control variables are always conflicting factors.

The potential function f /Eq. 13/ in these qualitative applications may be a likelihood function, a bimodal density function or a cost function /in an economic catastrophe model/.

The second group of examples contain mostly catastrophe models for mechanical movement where the state transition functions are given by solutions of well known differential equations. Use of special mathematical transformations which of these well-known state transition functions lead to catastrophe surfaces.

The third and smallest group of examples contains catastrophe models of various complex and poorly known processes such as phytoplankton growth and die off [4]. For these cases the modeling procedure is as follows:

. specifying the differential equations for modeling the investigated process

. transforming the differential equations into a form which is similar to the differential equation for a known catastrophe.

Considerations for further applications may be summed up as follows:

. The two control variables should be conflicting factors.

. The underlying function f which models the sudden state change may be a density function.

. A mathematical model common to a number of mechanical systems subject to sudden jumps is determined by two conflicting factors and one state variable; it is often represented by the cusp catastrophe model.

The steps necessary for achieving the goals mentioned in Section "Model Goals" are:

. Specification of the parameters /state variable, control variables, function f/ of the water-inrush process.

. Comparison of the mathematical model of the water inrush process with a catastrophe surface.

. Determination of the catastrophe function which yields the best numerical approximation to empirical water inrush data.

THE CUSP CATASTROPHE MODEL OF MINE WATER INRUSHES

The Variables

The state variable is the water inrush yield q or lg q. The two control variables are: the head s, and the minimum normal rock pressure σ_m. These two variables are conflicting factors. The other variables, such as layer thickness m and rock properties, are considered to be constant.

The Potential Function

The function f is the bimodal density function P(q) given in Eq. 12 and represented as Fig. 6/a or 6/b. This function f is determined empirically.

However, the rock-water mechanical interaction changes the parameters of all density functions by widening and closing the fissures in the protective layer [7, 8]. This is the reason for the difficulties in determining the catastrophe surface by minimizing the function f ($df/dq = 0$).

Comparison Between the Proposed Approach and Previous Models

The comparison of Fig. 4 and Fig. 5 appears that the mine water inrush process shows the four main features of the cusp catastrophe surface. These are: bimodality, sudden jump, bifurcation and hysteresis.

The hysteresis can be detected by laboratory experiments [7] and it also appears under mining conditions, when the increasing of the rock stress stops the small inrushes.

Let us present some other investigations on the qualitative similarities between the mine water inrush state surface and the cusp surface.

As hypothesized before, the water inrush process is represented by a rock stress variable o, a water movement variable s, and the state variable q /Fig. 4/. The state variable surface represents only the mean values of the yield q, and two mean value functions for the threshold gradient of the protective layer /one for opening, and the other for closing the fissures/. This schematic surface of the mean q values seems to resemble to a cusp catastrophe surface. We must see if the density function f exists.

Let us examine a surface section for a constant value of σ /Fig. 5/ with density functions P(q) of q being represented for some given value of s: the points where

$\frac{d\,P(q)}{dq} = 0$ give us a manifold curve similar to a section of the cusp catastrophe surface [9].

Using transformation a the variables

$$Q = \lg q \qquad S = \lg s \qquad /15/$$

the equation /12/ modified as follows:

$$\varphi(\lg q) = P_o(\lg q) \cdot [F'(s^*)] - P_w(\lg p) \cdot [1-F(s^*)] \qquad /16/$$

As mentioned before p(q) is lognormal distribution, consequently PQ is a normal one. The transformed curve

$$\frac{d[\varphi(Q)]}{dQ} = \frac{d[\varphi(\lg q)]}{\lg q} = 0 \qquad /17/$$

give better similarities to the mainfold curve of the cusp catastrophe model /Fig. 6/b/.

TRIALS FOR FIXING THE MODEL

Data Processing

As a result of a century long mining activity under heavy karstic water hazard, data for about 600 karstic water inrushes are availabee.

Original observations were taken on the yield of inrushes (q) on the location /x, y, z coordinates/ and mining conditions /longwall face, shortwall face, headway draining/. The original observations and the borehole data made it possible to determine the thickness and quality of the protective layer (m) and water head (s).

There were only a few direct measurements on the rock stress conditions, but records of mining depth protective layer thickness (m) and type of mining activity /size of openings/ made it possible to determine approximately the average value of the minimal vertical rock stress in the middle of the protective layer /between the mining opening and the reservoir rock/. Specifically, to accomplish this task, a finite element model was fitted to mining observations.

Data on Microseepage and Small Inflows

There were no direct observations on microseepage and very small inflows because this phenomenon was not important from the viewpoint of mining operations. These events were denoted as "nothing has happened" or "zero data". But the protective layer thickness (m), the water head (s), the mode of operations /size of openings/, the depth of the operations (z), are known everywhere, even when "nothing has happened". Consequently the average vertical rock stress may also be determined using the above mentioned finite difference model.

The possible number of "zero data" is given by Eq. /11/. The zero data are located in those mining fields, where karstic caves and fissures are covered by a protective layer which prevent water inrushes from occuring. Since the precise mine sites with "zero data" are not known to begin with, a simulation was run to assign the sites. A number N_o of inrushes given by Eq. /11/ was assumed. For each mining field the parameters (s, σ) of this field were used as parameters of zero data.

The data on yields (q)were also obtained from a statistical distribution of very small values of q. Two approaches were used for model fitting: first, conventional regression analysis and second, a multimodal density function.

The model used for regression analysis was the following parameterization of the cusp surface:

$$\delta(\bar{q} - \lambda)^3 + a(\overline{\Delta s}, \sigma)(\bar{q} - \lambda) + b(\overline{\Delta s}, \sigma) = 0 \qquad /18/$$

$$\begin{bmatrix} a(\overline{\Delta s}, \bar{\sigma}) \\ b(\overline{\Delta s}, \bar{\sigma}) \end{bmatrix} = \begin{bmatrix} c_{11} \\ c_{21} \end{bmatrix} + \begin{bmatrix} c_{12} & c_{13} \\ c_{22} & c_{23} \end{bmatrix} \begin{bmatrix} \overline{\Delta s} \\ \bar{\sigma} \end{bmatrix} \qquad /19/$$

Here $\bar{q}$, $\overline{\Delta s}$, and $\bar{\sigma}$ are q, Δs, and σ normalized to zero mean and unit variance to preserve computational accuracy. The parameter λ is the "location" parameter and indicates where the cusp singularity occurs. The constant δ is a concentration parameter, and indicates how concentrated the data is in the vicinity of the cusp surface. The larger δ, the more concentrated the data. Negative and zero values of indicate that the cusp model is inappropriate for the data [2] . Note that this model is similar to that discussed in [9], except the trigonometric functions have been replaced by constants and the location and concentration parameters have been included.

Although no "goodness of fit" tests exist at present for statistical estimates using catastrophe models, inferences can be drawn from the mean square of the difference between the data points and the estimated inrush /mean square error/. In addition, a small or negative value of will indicate that the cusp model was not appropriate for the data.

In Table 1, the means, variances, and number of data points for all the mines and the total TATA, total DOROG and total mines regressions are listed. In Table 2, the corresponding mean square errors for all the mines considered are listed, as well as λ and δ.

The second approach used to analyze data leans on the fact that the system function f is given by a pdf in Eq. 13. We propose to use functions of the form

$$f^{*} = K_e \exp\left(-\gamma_4\left(x^4 - 2bx^2 - 4ax\right) / \varepsilon\right) \qquad /20/$$

ε = variance $\qquad$ K_e = normalizing constant

which can be used to describe a process thought to be governed by a cusp probability law [2]. Unfortunately, however, statistical estimators for these processes assume constant values of b and a, which in the case of the minewater inrush data is obviously not occurring.

For these cases, b and a would be parameters for the distribution, similar to the mean and variance for the standard normal on the location and shape parameters of the gamma.

Though the computation of the second approach has not completely finished yet, an analytical presentation of the fold curve concerning the surface q-s is also presented based on the equations /16/ and /17/.

$$\frac{\partial\left[P(\lg q)\right]}{\partial(\lg q)} = P_W'(\lg q)\,F(s_o) + P_o'(\lg q)\left[1 - F(s_o^{*})\right]$$

$$+ F'(s_o^{*}) \cdot P_W(\lg q) + \left[1 - F(s_o^{*})\right]' P_o(\lg q) = 0 \qquad /21/$$

$$F(s_o^{*}) = \left[\int_{-\infty}^{s} \frac{1}{120\pi} e^{-(170-t)^2} dt\right] \left(\frac{s}{1000} - \frac{s}{100}\right) \qquad /22/$$

$$P_o(q) = \frac{1}{1.2\,\pi} e^{-\left[\frac{\lg s + 6 - \lg q}{1.2}\right]^2} \qquad /23/$$

$$P_W(q) = \frac{1}{1.2\,\pi} e^{-\left[\frac{\frac{1}{2}\lg s - 1 - \lg q}{1.2}\right]^2} \qquad /24/$$

The above /22/ and /24/ equations are based on the empirical data of DOROG coalfield, Eq. /23/, the distribution of microseepage is an estimated function.

The Eq. /22/ can be expressed in form of an approximated polinom.

Let us briefly present two possible practical applications of this remark:

1. According to the existing Hungarian Mining Safety Regulations, the size of the water barrier pillars must be determined by using a preset value of the threshold hydraulic gradient /not depending on the rock stress conditions/. This regulation value had been determined by empirical data on mining activity at a depth of 100 to 200 meters [10]. The procedure leads to extremely large sizes of water barrier pillars at depth of 400 to 500 meters where new mines are going to be opened. The Eq. /4/ points the way to reducing the size of water barrier pillars at the greater mining depth.

2. In some Hungarian coalfields /Balinka, Dudar, Varpalota/ the layers located between the coal layers and a lower karstic limestone aquifer consist of a series of impermeable clays and water-bearing sands, as shown in Fig. 8/a.

The karstic water inrushes are often initiated by inrushes from nearest water-bearing sand /in Fig. 8/. The sudden drop of the water pressure in aquifer a_1 /caused by the water inrush/ increases the differential rock pressure between the aquifers a_1 and a_2, and a water inrush occurs from the aquifer a_2. The process may continue step by step, and finally a karstic water inrush will have occurred.

For cases when a preliminary lowering of the karst water level is not possible because of the necessity to protect karstic water balance, the only was of reducing the risk of karstic water inrush may be to control the water pressure in the water-bearing sands.

According to the former approach - having supposed that the threshold gradient is not dependent on the rock stress - that is, J_O = const, the optimal rate of water level lowerint in sands has to be determined by the equation:

$$J_{a_1} = J_{a_2} = J_{a_3}$$

/see Fig. 8/b/, because under these conditions the rikk of water inrush from either one of the aquifers is constant.

Taking into account that the threshold gradient of the protective layers depends on the rock stress, that is, $J_{O_{a_1}}$ $J_{O_{a_2}}$ $J_{O_{a_3}}$ as indicated in the $_{min}$ curve in

$$F(s_o^*) = \frac{\frac{s}{1000} - \frac{1}{100}}{\sqrt{2\pi} \cdot 60n} \sum_{n=0}^{\infty} \left(\frac{s - 170}{6\sqrt{2}} \right)^{2n+1} \frac{(-1)^n}{n! \cdot (2n+1)} \qquad /25/$$

using the Eqs. /23/, /24/ and /25/. The Eq. /21/ is expressed as follows:

$$\frac{\frac{s}{1000} - \frac{1}{100}}{60n\sqrt{2\pi}} \sum_{n=0}^{\infty} \left(\frac{s - 170}{6\sqrt{2}} \right)^{2n+1} \cdot \frac{(-1)^n}{n!\,(2n+1)} \cdot \left\{ \frac{lgs + 6 - lg\,q}{0.6\sqrt{2\pi}} \cdot e^{\frac{\left(\frac{1}{2} lgs - 1 - lgq\right)^2}{0.72}} + \right.$$

$$\left. \frac{lgq - lgs + 6}{0.6\sqrt{2}} \cdot \frac{1}{0.1\sqrt{2\pi}} e^{\frac{\left(\frac{1}{2} lgs - 1 - lgq\right)^2}{0.72}} \right\} -$$

$$\cdot \frac{lgq - lgs + 6}{0.6 \cdot \sqrt{2}} \cdot \frac{1}{0.6\sqrt{2\pi}} e^{\frac{(lgs - 6 - lgq)^2}{0.72}} \qquad /26/$$

Though Eq. /26/ illustrates the difficulties of the analytical transformation of this equation into the canonic polinom of the fold curve given by the cusp catastrophe surface, the Fig. 7 also shows the Eq. /26/.

It is a fold curve indeed, which seems to be similar to a canonic form of the fold curve. Perhaps more other transformations should be made for a better model fitting.

Though a cusp surface of the simplified model of the mine water inrushes has been presented, and the mathematical difficulties of the model fitting may be eliminated, some practical difficulties are also existing . These are as follows:

. There are no direct observations on the microseepage /zero data/.

. The data on rock stress are only estimated ones.

. The indicated and estimated data are concerning only a relatively small area of the process space.

SOME PRACTICAL BYPRODUCTS

Investigations presented so far represent only the first steps of the catastrophe modeling approach. Yet, certain features may already be used for a qualitative design of the process control system.

The approach presented in Sections "Approximate Model of the Mine Water Inrush Process" and "A Simplified Two-Phase Model" pointed out that the threshold gradient of the protective layer depends on the rock stress /Eq. 4/. Consequently, the protective layer thickness and water barrier pillar thickness must not be determined by the same value if the threshold gradient of rock stresses are varying.

Fig. 7c, the equal /and minimal/ risk may be reached under the conditions:

$J_{a_1} \quad J_{a_2} \quad J_{a_3}$

Comparing the two conditions, it can be seen that the new approach determines such rates of water level lowering that give us a lower risk of karstic water inrush. On the base of this approach [9] proposals were prepared on increasing the efficiency of mine water control for Varpalota coalfield.

It is not claimed that only the approach presented herein can lead to this more effective control method, however, the practical applications presented above provide an example of bifurcation theory approach leading to practical qualitative consequences sooner than any other possible approach.

CONCLUSIONS

The qualitative similarities between the probabilistic catastrophe model and the two-phase model of mine water inrushes have appeared.

The fitting of the canonic cusp surface to the empirical data of mine water inrushes shows difficulties which are as follows:

. abscence of indicated data on microseepage /"zero data"/

. the data on rock stress conditions are only estimated

. the available empirical data are concerning a relatively small area of the process space.

The phenomenological analysis on the two-phase process of the mine water inrushes concerning the bifurcation models gives some practical profils.

Though earlier experiments show dependence between the rock stress and the threshold hydraulic gradient of the protective layer, the practical consequences mentioned in Section "Some Practical Byproducts" have appeared for the author as a consequence of the analysis presented in this paper.

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Table 1 - Means and Variances of Control and State Variables for Mines Estimated Using Regression

Mine	q Mean	q Variance	Δs Mean	Δs Variance	σ Mean	σ Variance	N
TATA3	1.85	30.12	7.86	3.45	3.54	4.20	13
TATA6	0.82	2.04	7.44	4.29	6.66	9.43	61
TATA14	0.87	1.60	7.03	4.69	8.37	14.35	52
TATASI	0.84	10.49	7.13	2.99	6.69	6.67	59
ALL TATA's	2.03	80.37	7.14	4.12	6.90	11.09	195
DORO6	1.98	61.16	17.30	42.72	17.90	80.34	93
DORO10	0.86	2.04	8.89	8.10	15.64	11.30	11
DORO12	1.13	2.40	28.30	10.61	35.86	74.21	44
DORO15	11.47	666.06	29.09	1.66	31.67	159.00	9
DORO17	2.46	11.13	14.56	25.51	9.13	17.98	23
DORO19	0.16	0.04	37.72	8.07	57.64	87.13	21
DOROCS	2.89	54.40	7.97	17.33	12.74	31.41	54
DOROEB	4.32	73.00	4.01	6.81	4.86	4.98	29
DOROER	0.64	1.02	27.78	0.72	39.88	10.27	8
DOROTA	3.49	44.53	3.57	5.70	5.95	7.68	42
ALL DORO's	2.56	57.21	16.11	126.54	19.74	247.99	346
ALL Mines	2.46	67.93	11.87	78.71	13.40	129.13	520

Table 2 - Estimated Parameters and Mean Square Errors

Mine	λ	δ	MSE
TATA3	1.85×10^{-1}	3.77×10^{7}	0.729
TATA6	1.29	1.37×10^{1}	3.07
TATA14	1.05	6.90	2.41
TATASI	7.60×10^{-1}	5.95×10^{3}	2.18
ALL TATA's	0.379	2.83×10^{3}	1.03
DORO	6.86×10^{-1}	2.85×10^{3}	-
DORO10	5.90×10^{-1}	1.09×10^{3}	2.10
DORO12	9.49×10^{-1}	2.51×10^{1}	2.14
DORO15	2.62×10^{-2}	2.17×10^{9}	0.193
DORO17	3.86×10^{-1}	6.89×10^{2}	1.15
DORO19	5.15	4.92×10^{-3}	7.67
DOROCS	3.95×10^{-1}	6.23×10^{3}	0.814
DOROEB	1.25×10^{-1}	2.31×10^{5}	0.500
DOROER	3.33×10^{-1}	4.66×10^{3}	2.04
DOROTA	3.29×10^{-1}	4.80×10^{3}	0.941
ALL DORO's	5.71×10^{-1}	1.57×10^{3}	1.33
ALL Mines	6.33×10^{-1}	1.46×10^{3}	1.43

List of Figures

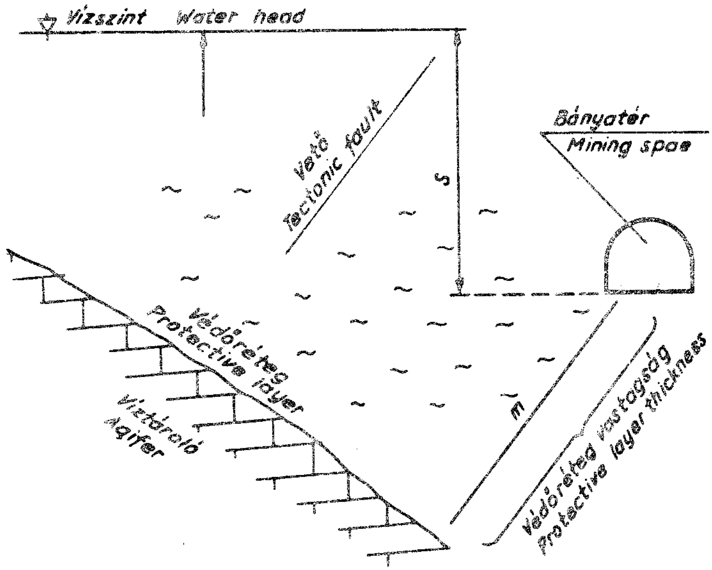
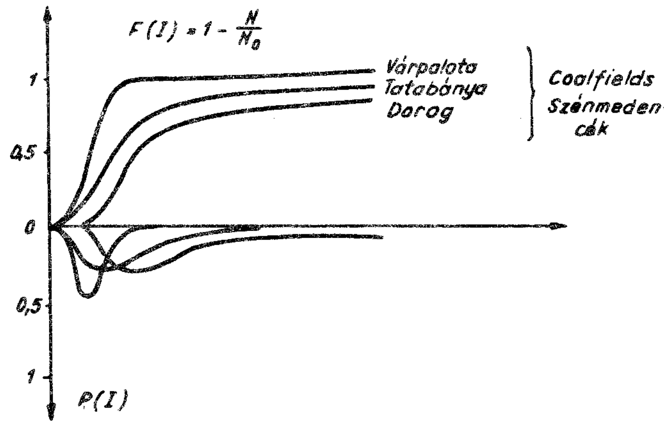

Vízszint Water head
Vető
Tectonic fault
S
Bányatér
Mining spae
Védőréteg
Protective layer
Víztároló
Aqifer
m
Védőréteg vastagság
Protective layer thickness

Fig. 1. ábra



F(I) = 1 - N/N0
1
0.5
0
0,5
1
P(I)
Várpalota
Tatabánya
Dorog
Coalfields
Szénmedencék

Fig. 2. ábra

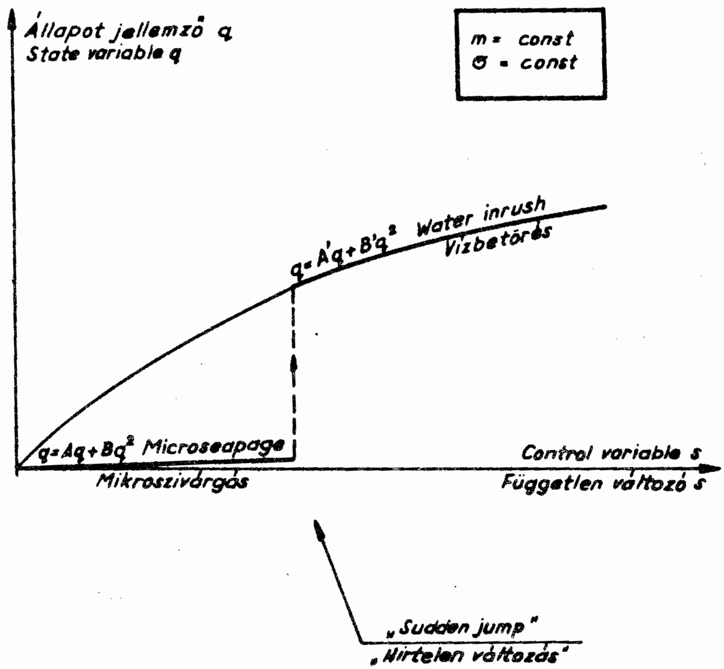

Állapot jellemző q
State variable q
m = const
σ = const
q = A'q + B'q² Water inrush
Vízbetörés
q = Aq + Bq² Microseapage
Mikroszivárgás
Control variable s
Független változó s
„Sudden jump"
„Hirtelen változás"

Fig. 3. ábra

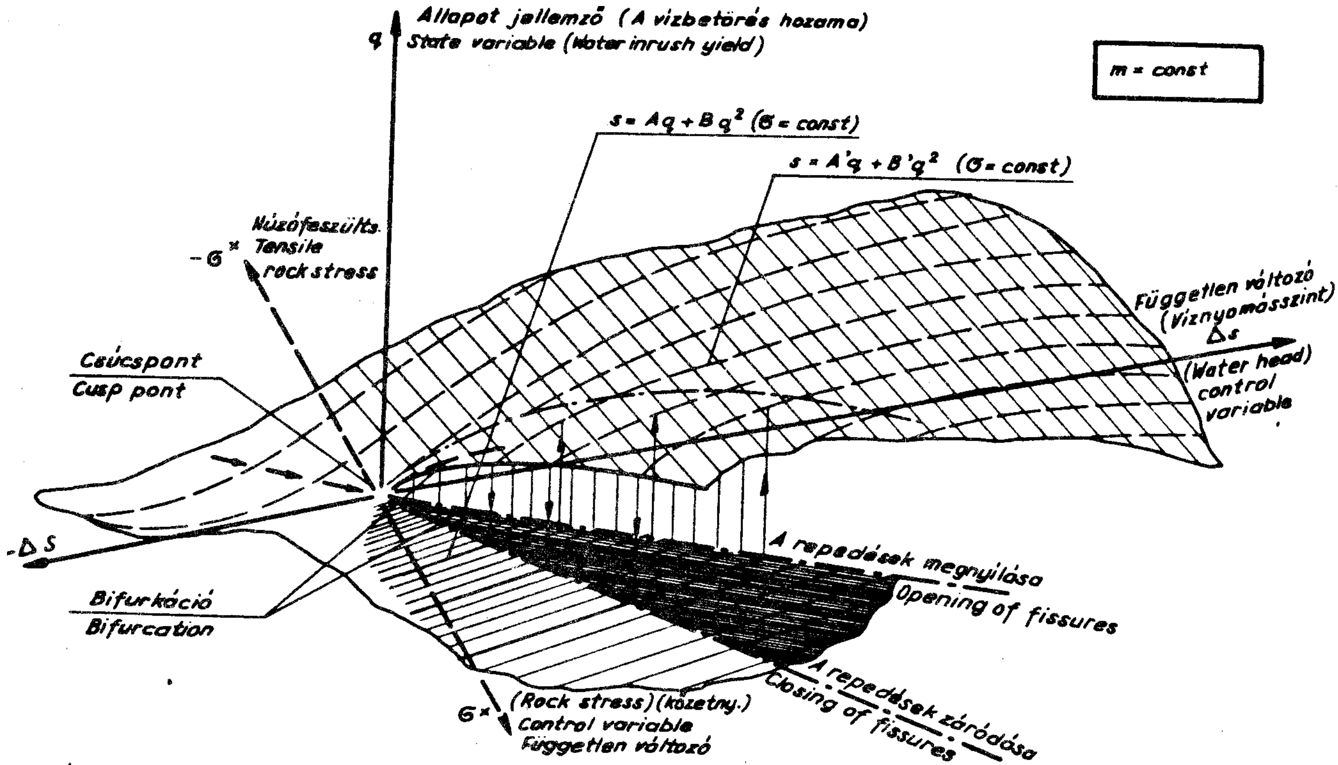

Állapot jellemző (A vízbetörés hozama)
q State variable (Water inrush yield)
m = const
s = Aq + Bq² (σ = const)
s = A'q + B'q² (σ = const)
Húzófeszülts.
-σ* Tensile
rock stress
Csúcspont
Cusp pont
Független változó
(Víznyomásszint)
Δs
(Water head)
control
variable
-Δs
A repedések megnyílása
Opening of fissures
Bifurkáció
Bifurcation
A repedések záródása
Closing of fissures
σ* (Rock stress) (kőzetny.)
Control variable
Független változó

Fig. 4. ábra

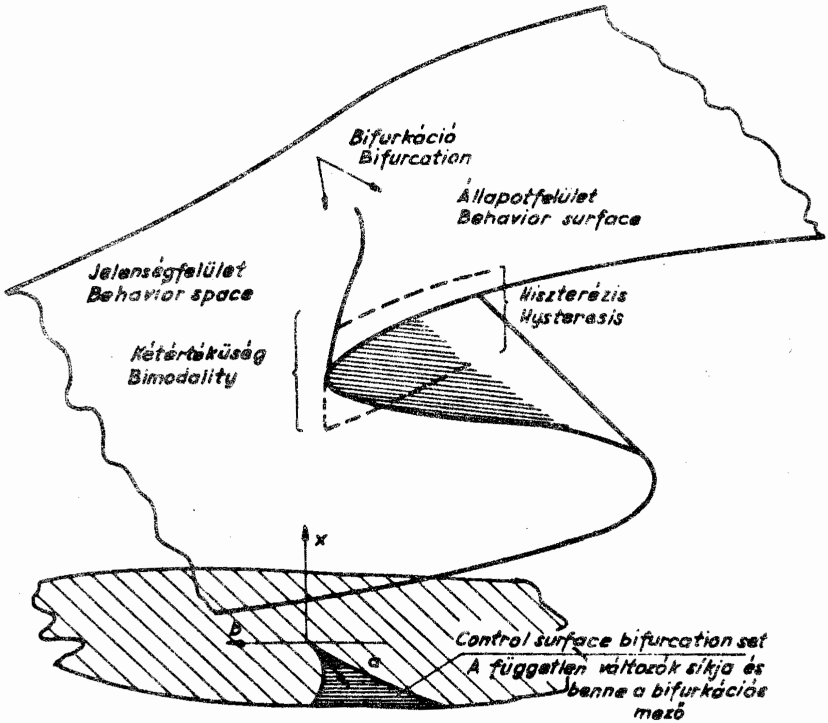

Bifurkáció
Bifurcation
Állapotfelület
Behavior surface
Jelenségfelület
Behavior space
Hiszterézis
Hysteresis
Kétértékűség
Bimodality
x
b
a
Control surface bifurcation set
A független változók síkja és
benne a bifurkációs
mező

Fig. 5. ábra

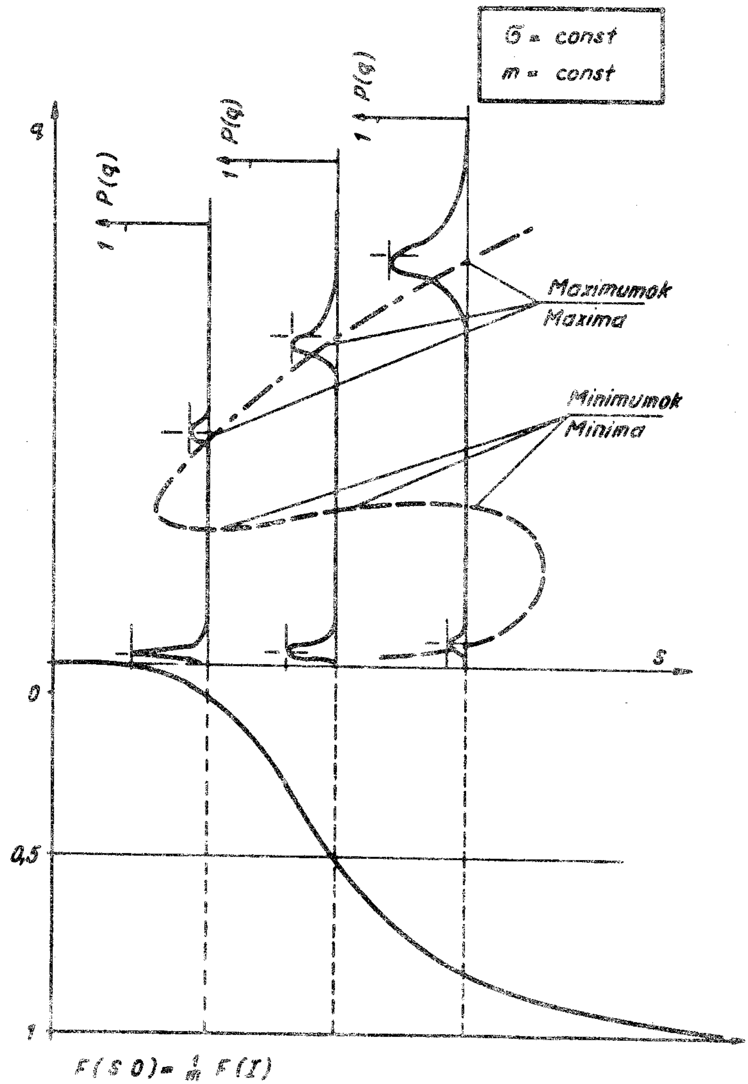

σ = const
m = const
q
1 P(q)
1 P(q)
1 P(q)
Maximumok
Maxima
Minimumok
Minima
S
0
0,5
1
F(S 0) = 1/m F(I)

Fig. 6. ábra

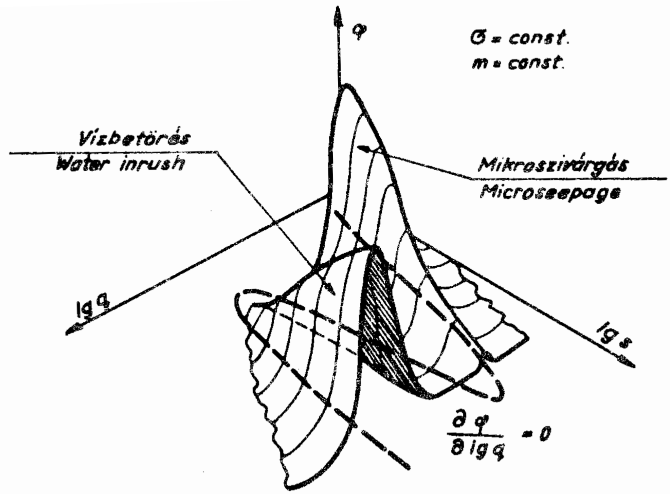


Fig. 6/b. ábra

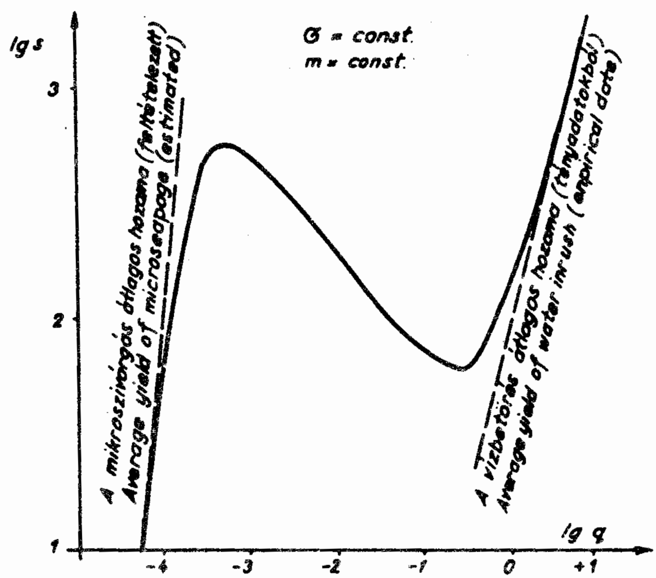


Fig. 7. ábra

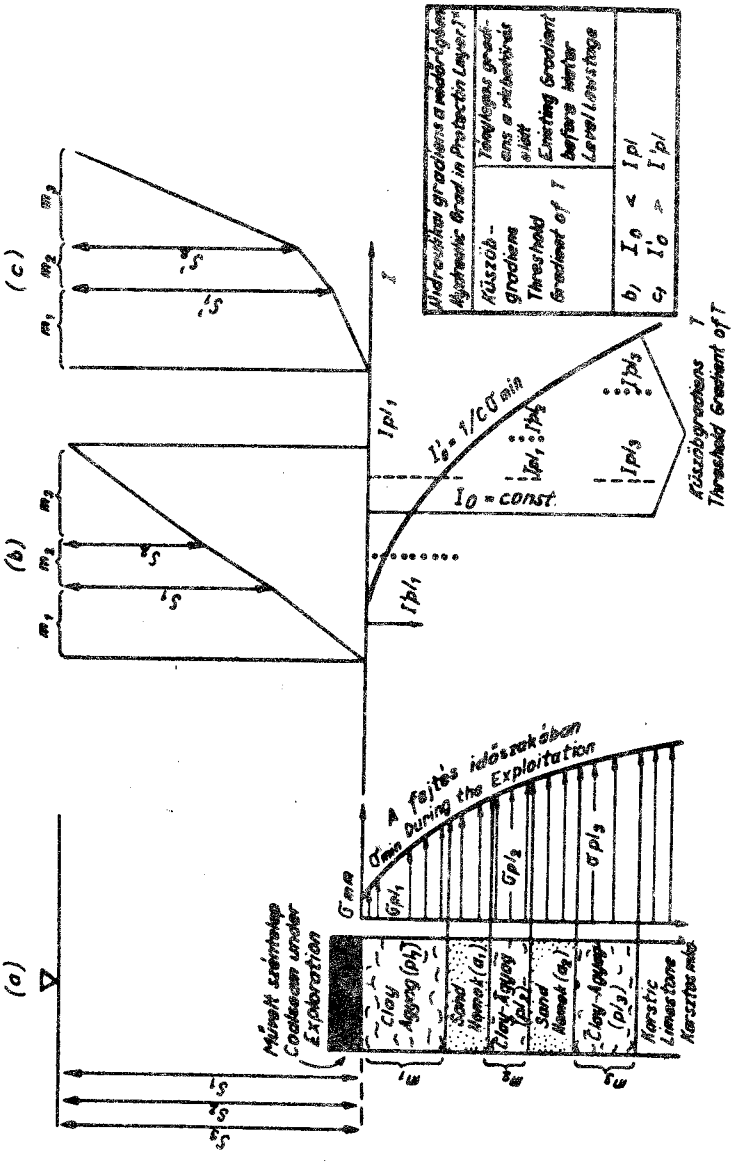

(a)
(b)
(c)
Művelt széntelep
Coalseam under
Exploration
Clay
Agyag (pl1)
Sand
Homok (a1)
Clay-Agyag
(pl2)
Sand
Homok (a2)
Clay-Agyag
(pl3)
Karstic
Limestone
Karsztos mész
A fejtés időszakában
σmin During the Exploitation
I0 = const
Küszöbgradiens I
Threshold Gradient of I
Hidraulikai gradiens a védőrétegben
Hydraulic Grad in Protection Layer I
Küszöb-
gradiens
Threshold
Gradient of I
Tényleges gradiens a vízbetörés előtt
Existing Gradient
before Water
Level Lowering
b) I0 < Ipl
c) I0 > I'pl

Fig. 8. ábra