ANALYSIS OF PLANE GROUNDWATER FLOW BY THE BOUNDARY INTEGRAL EQUATION METHOD

J.O. Watson and E.T. Brown

Department of Mineral Resources Engineering
Imperial College of Science & Technology,
London SW7 2BP, U.K.

ABSTRACT

Numerical models are required to make calculations of the distributions of potential and the flows into excavations for most mining related groundwater flow problems. The formulation of such problems using the boundary integral equation method and the implentation of this formulation in a computer program, GFLOW, are described. This program is designed to solve problems involving plane, unconfined flow in homogeneous rock masses having anisotropic permeabilities determined, for example, by flow through major joint sets. An iterative procedure is developed for determining the location of the phreatic surface in the steady state condition. Although boundary elements with quadratic functional variation and advanced numerical procedures are used, GFLOW has been designed so that it can be used with computer systems as small as a 64K byte microprocessor, given a hard disc on which to hold scratch files. The solution is given for an illustrative problem involving flow through an anisotropic rock mass into a single horseshoe shaped tunnel.

INTRODUCTION

In feasibility and planning studies for mining operations in water-bearing ground, it is important to be able to make predictions of the likely rates of groundwater inflow into the mining excavations and of the positions of the phreatic surface at various stages of mining. Such predictions permit estimates to be made of the costs of draining the mine, the capacities of the pumping equipment required, the likely extent of operational difficulties caused by water inflows, and the effects of drawdown on surface installations and groundwater supplies.

In order to be able to make these predictions, knowledge is required of the regional geohydrology, including initial piezometric levels and recharge sources, the geological structure and hydraulic characteristics of the rock mass surrounding the mine, and the geometry of the proposed excavations. Most importantly, a method of analysis is required. Many occurrences of water in mines are extremely difficult to predict and analyse. These include inrushes from caverns in carbonate rocks or from isolated pockets of water-storing rock and flows through conduits or from sources that are at least partially man-made [1].

In general, however, flows will be through the primary permeability of the rock itself, through the secondary permeability due to the joints in the rock mass, or through major geological conduits such as faults and dykes. Inflows through, or controlled by, major geological conduits have caused serious mining problems in the past [1,2]. Provided their existence, hydraulic characteristics and recharge sources can be predetermined. flow through such features can be analysed using the finite element method [3], for example.

Excluding special features such as those referred to above, the seepage of water in a rock mass will be typically through the joints or discontinuities rather than through the blocks of intact rock [4]. Only in very porous rocks, such as some sandstones and limestones, will the primary permeability of the rock be dominant. On the scale of a mine, the joints will be very numerous, and so it will be impracticable to determine their distributions and individual characteristics and to consider the flow through each of them in the analysis. If the joint spacing is small compared with the dimensions of the problem domain, it is acceptable to treat the rock mass as an equivalent continuum with permeabilities such that, in the large, the hydraulic characteristics of the continuum and the jointed rock mass are equivalent [4,5]. Generally the permeability of the equivalent continuum will be anisotropic and it may be necessary to treat the rock mass as being composed of a number of regions each with different characteristics. In the analysis presented herein, the rock mass may be anisotropic but is considered homogeneous.

To calculate the variation of potential through the continuum and the flow across any part of its boundary, it is necessary to solve a boundary value problem. For some simple problems, generally involving cylindrical excavations, closed-form solutions have been obtained for confined and unconfined flows [5]. These solutions have been adapted to give simple, and approximate, predictions of inflows into underground excavations [6, 7]. In general, closed-form solutions can only be obtained for cases involving linear flow laws and excavation geometries and boundary conditions which can be described by simple functions. In other cases, numerical methods must be used.

NUMERICAL METHODS

Boundary value problems of groundwater flow are usually solved by finite difference or finite element methods [8,9], in which it is necessary to define a grid or mesh throughout the region of interest, and to construct and solve a system of simultaneous equations in terms of unknowns associated with node points distributed both inside the rock mass and on its surface. If there is a phreatic surface, this system must be solved many times during an iterative calculation of the location of that surface. Since the order of the system is large, computing costs are high. In addition, the governing partial differential equation is not exactly satisfied at each point of the continuum, and so the solution obtained corresponds to a residual distribution of sources and sinks throughout the rock mass.

Boundary integral methods are alternatives to finite differences and finite elements, in which the partial differential equation is transformed to a boundary integral equation [10]. To solve the integral equation, a mesh of elements is defined on the surface only of the region of interest, and a system of equations in terms of unknowns associated with nodes on the surface only is constructed and solved. The system of simultaneous equations is smaller, and the solution obtained satisfies exactly the governing partial differential equation at every point of the continuum.

The integral equation

Let us consider the three dimensional problem. Let $\alpha(y)$ and $\beta(y)$ be arbitrary twice continuously differentiable potential fields, and let $v_i(\alpha)$ and $v_i(\beta)$ be the corresponding fluid velocities. Then by making suitable substitutions in the divergence theorem[10], it can be shown that

$$\int_V \{\alpha(y)\,\frac{\partial v_s(\beta)}{\partial y_s} - \beta(y)\,\frac{\partial v_s(\alpha)}{\partial y_s}\}dV_y = \int_S \{\alpha(y)\;v_s(\beta) - \beta(y)\;v_s(\alpha)\}.\,n_s(y)\;dS_y \qquad (1)$$

where S is the surface of the region V, and $n_s(y)$ is the unit outward normal to S at the point y. Equation (1) is analogous to Betti's reciprocal theorem of elasticity[10], and, for an isotropic continuum of unit permeability, reduces to Green's symmetric identity. Let us take $\alpha(y)$ to be the solution $u(y)$ of the boundary value problem, and let $\beta(y)$ be the potential $U(x,y)$ which would arise in an infinite region if there were a unit source of fluid at the point x on S. In order to satisfy the conditions of differentiability, let us exclude the singularity of $U(x,y)$ at x by writing equation (1) for the region $V - v(x,\varepsilon)$ where $v(x,\varepsilon)$ is that part of V which lies within a sphere of radius ε centred at x (see Fig.1). Then because $u(y)$ and $U(x,y)$ satisfy the governing partial differential equation everywhere in $V - v(x,\varepsilon)$ the volume integral vanishes and

$$\int_{S-S(x,\varepsilon)\,+\,s(x,\varepsilon)} \{u(y)\;v_s(U) \;-\; U(x,y)\;v_s(u)\}\;\;n_s(y)\;dS_y \;=\; 0 \qquad (2)$$

where (see Fig.1) $S(x,\varepsilon)$ is that part of S which lies within the sphere of radius ε and $s(x,\varepsilon)$ is that part of the surface of the sphere which lies within V. Now let $\varepsilon \to 0$. It can be shown that, in the limit[10],

$$c(x)\;u(x) \;+\; \int_S T(x,y)\;u(y)dS_y = \int_S U(x,y)\,t(y)dS_y \qquad (3)$$

where $t(y)$ and $T(x,y)$ are the inflows across S at y due to the potential fields $u(y)$ and $U(x,y)$, and $c(x)$ is a known function of x. If the tangent plane is continuous at x, $c(x) = \frac{1}{2}$. Equation (3) is the boundary integral equation of the direct formulation. If either $u(y)$ or $t(y)$ is known at every point y of S, then this equation can be solved for whichever of $u(y)$ and $t(y)$ is unknown. If results are required at points x in V, they can then be computed using the results

$$u(x) = \int_S U(x,y)\,t(y)dS_y - \int_S T(x,y)u(y)dS_y \qquad (4)$$

and

$$v_i(x) = \int_S D_i(x,y)\,t(y)dS_y - \int_S S_i(x,y)u(y)dS_y \qquad (5)$$

which, like the boundary integral equation, are derived from equation (1).

For the plane problem, the analysis is the same as for three dimensions, except in that the region $v(x,\varepsilon)$ is taken to be part of a disc of radius ε rather than of a sphere. For the plane problem in which the principal directions are parallel with the coordinate axes,

$$U(x,y) = \frac{1}{2\pi\sqrt{k_1 k_2}} \log\frac{1}{\rho}$$
$$T(x,y) = \frac{(x_s - y_s)\, n_s(y)}{2\pi\rho^2 \sqrt{k_1 k_2}} \tag{6}$$

where k_i are permeabilities, x_s and y_s are the coordinates of the points x and y, $n_s(y)$ is the unit outward normal to S at y, and

$$\rho = \sqrt{\frac{(x_s - y_s)^2}{k_s}} \tag{7}$$

In equation (5)
$$D_i(x,y) = -k_i \frac{\partial}{\partial x_i} \{U(x,y)\}$$
$$S_i(x,y) = -k_i \frac{\partial}{\partial x_i} \{T(x,y)\} \quad \text{(no sum on i)} \tag{8}$$

THE GROUNDWATER FLOW PROBLEM

Equations (3), (4) and (5) are valid if S is taken to be the boundary of that part of the rock mass which is saturated, this being the rock below the phreatic surface. The location of the phreatic surface is not known in advance, and so the solution of a problem of groundwater flow is an iterative process. One method [10,11] is to solve equation (3) on the assumption that the rock is saturated everywhere, compute a first estimate of the phreatic surface (e.g. the surface on which potential equals altitude), solve equation (3) for the rock mass below that surface, compute a second estimate and so on until changes in the computed location of the phreatic surface are sufficiently small. The repeated equation solution is expensive; worse, the algorithm is not robust because if the computed phreatic surface intersects some other part of the boundary S, such as the crown of a tunnel, then the integral equation is no longer soluble. An alternative iterative method in which the boundary S does not move from its initial position is therefore proposed here.

Let us take S to be the boundary of the entire mass of rock under consideration, including rock that may be dry, and for the purpose of writing equation (3) suppose that over the part S^1 of S which may be wet or dry, inflow $t(y)$ is known and potential $u(y)$ unknown. Let $u^{(n)}(y)$ and $t^{(n)}(y)$ be the nth iterated values of potential and inflow at y. The iteration is begun by taking $t^{(1)}$ to the zero on S^1, and solving equation (3). Let us denote by $S_d^{(n)}$ that part of S^1 on which $u^{(n)}(y)$ is less than altitude, and by $S_w^{(n)}$ the rest of S^1. Below $S_d^{(n)}$, the nth iterated phreatic surface is taken to be the surface on which computed potential equals altitude. During the iteration, the following adjustments are made to inflow on S^1. At a point y on $S_d^{(n-1)}$ below which there is a point on the (n-1)th iterated phreatic surface,

$$t^{(n)}(y) = t^{(n-1)}(y) - ka^{(n-1)}(y) \tag{9}$$

where $s^{(n-1)}(y)$ is the computed inflow across the (n-1)th iterated phreatic surface at the point below y, and k is a relaxation factor. For y on $S_w^{(n-1)}$,

$$t^{(n)}(y) = \min \{ [t^{(n-1)}(y) - k\Delta t(y)] , 0\} \qquad (10)$$

where $\Delta t(y)$ are the adjustments to inflow on $S_w^{(n-1)}$ that would be required to set potential equal to altitude on $S_w^{(n-1)}$. The iteration is terminated when inflow $s^{(n)}(y)$ across the phreatic surface, and differences between potential and altitude on $S_w^{(n)}$, are sufficiently small.

Numerical analysis

Let us represent the boundary S by p elements S_b, each with three nodes (see Fig.2). Let $x_i(b,c)$ $u(b,c)$ and $t(b,c)$ be cartesian coordinates of, and potential and inflow at, node c of element S_b. Then the coordinates of, and potential and inflow at, an arbitrary point of element S_b are given in terms of the shape functions $N^c(\xi)$ of the intrinsic coordinate ξ by

$$\begin{aligned} x_i(b,c) &= \sum_{c=1}^{3} N^c(\xi)x_i(b,c) \\ u(b,c) &= \sum_{c=1}^{3} N^c(\xi)u\ (b,c) \qquad (11) \\ t(b,c) &= \sum_{c=1}^{3} N^c(\xi)t\ (b,c) \end{aligned}$$

where [10]

$$\begin{aligned} N^1(\xi) &= \tfrac{1}{2}\ \xi(\xi+1) \\ N^2(\xi) &= 1-\ \xi^2 \qquad (12) \\ N^3(\xi) &= \tfrac{1}{2}\ \xi(\xi-1) \end{aligned}$$

and ξ varies from -1 to +1. Let there be a total of q nodes x^a on S, the number of node c of element S_b being d(b,c). Then a system of simultaneous equations in terms of potential and inflow at these nodes, approximating to the boundary integral equation, may be written by taking the point x of equation (3) to be located at each of the q nodes in turn and substituting the parameteric representations of equation (11):

$$c(x^a)u(x^a) + \sum_{b=1}^{p}\sum_{c=1}^{3} u(x^{d(b,c)}) \int_{S_b} T(x^a,y(\xi))N^c(\xi)J(\xi)\ d\xi$$
$$= \sum_{b=1}^{p}\sum_{c=1}^{3} t(b,c) \int_{S_b} U(x^a,y(\xi))\ N^c(\xi)J(\xi)\ d\xi \qquad (13)$$

where $J(\xi)$ is the jacobian $ds/d\xi$ where s = arc length, and the superscript a ranges from 1 to q. The integrals of kernel-shape function products appearing in equation (13) may be evaluated using Gaussian quadrature formulae [10], and known values of potential and inflow substituted to yield a system of q simultaneous equations in terms of q unknown nodal values, one per node. Where potential is given on both elements adjacent to a corner of the region under consideration, certain approximations must be made to reduce the number of unknowns associated with the node at the corner to one, but the resulting loss of accuracy is negligible except near the corner.

At the nth iteration, let us rearrange the numbering of the nodes so that nodes inside S^1 and on $S_w^{(n-1)}$ are numbered from 1 to r, nodes inside S^1 and on $S_d^{(n-1)}$ are numbered r+1 to s, and nodes inside $S-S^1$ or on the boundary between S^1 and $S-S^1$ are numbered from s+1 to q. Then equation (13) may be re-written

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} u_w^{(n)} \\ u_d^{(n)} \\ f^{(n)} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \begin{bmatrix} t_w^{(n)} \\ t_d^{(n)} \\ g \end{bmatrix} \qquad (14)$$

where A and B are matrices of known coefficients (A_{ij} and B_{ij} being sub-matrices), $u_w^{(n)}$ and $t_w^{(n)}$ are potential and inflow at nodes 1 to r, $u_d^{(n)}$ and $t_d^{(n)}$ are potential and inflow at nodes r+1 to s, and $f^{(n)}$ and g are unknown and known parameters at nodes s+1 to q. Premultiplying by A^{-1}

$$\begin{bmatrix} u_w^{(n)} \\ u_d^{(n)} \\ f^{(n)} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} t_w^{(n)} \\ t_d^{(n)} \\ g \end{bmatrix} \qquad (15)$$

where $C = A^{-1}B$. At nodes 1 to r, $t_w^{(n)}$ is given by equation (10), in which $\Delta t(y)$ is the adjustment to inflow on $S_w^{(n-1)}$ required to set potential equal to altitude on $S_w^{(n-1)}$. Let Δt_w be the vector of adjustments at nodes 1 to r. Then

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} \Delta t_w \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \Delta u_w \\ \Delta u_d \\ \Delta f \end{bmatrix} \qquad (16)$$

where $\Delta u_w = -(u_w^{(n-1)} - \text{altitude})$ at nodes 1 to r. Then

$$\Delta t_w = C_{11}^{-1} \Delta u_w \qquad (17)$$

At nodes r+1 to s, the point at which $u^{(n-1)}(x)$ equals altitude below each node is located by a Newton-Raphson iteration in which equations (4) and (5) are used to compute potential and flow. Equation (4) is then used to determine the slope of the phreatic surface, and then to calculate the inflow $s^{(n-1)}$ across it, so that $t_d^{(n)}(y)$ can be calculated according to equation (9).

The procedure, then, is in principle as follows:

1) set $t_w^{(1)} = t_d^{(1)} = 0$ and compute $u_w^{(1)}$, $u_d^{(1)}$ and $f^{(1)}$ according to equation (15)

2) for $n = 2,3,4 \ldots$ a) compute Δt_w from equation (17), then $t_w^{(n)}$ from equation (10)

b) locate the (n-1)th phreatic surface by Newton-Raphson iteration, then compute $t_d^{(n)}$ from equation (9).

c) compute $u_w^{(n)}$, $u_d^{(n)}$ and $f^{(n)}$ according to equation (15).

3) terminate the iteration when $s^{(n-1)}$ and Δu_w are acceptably small.

IMPLEMENTATION

The algorithm described in the preceding section is implemented in program GFLOW, this being a program designed primarily as a teaching and research facility, rather than as a means of solving practical problems as efficiently as possible. It is for that reason that boundary elements with quadratic functional variation are chosen instead of Hermitian cubic elements [12]. The logic of GFLOW is further simplified by taking the same order of Gaussian quadrature formula for all elements and positions of the first argument of the kernel, rather than varying the order according to the estimated rapidity of variation of the integrand [10]. Basic features of earlier programs for elastostatic analysis are however retained: about half the code is pre-processing, giving the user complete freedom of node and element numbering (the partitioning described in the preceding section is notional), considerable freedom of order of presentation of input data, automatic data generation facilities and readily comprehensible error messages; there is no known way of causing abnormal termination of execution other than by providing insufficient system resources; and the overlay structure and extensive use of scratch files allows the program to be run on small systems. GFLOW solves problems in which the boundary is represented by up to 100 elements (order q of matrix C of equation (15) up to 200) in 16K words of CDC central memory (1 word = 60 bits), and could be converted to solve problems of nearly that size on a 64K byte microprocessor, given a hard disc on which to hold scratch files. A simplified flow chart is shown in Fig.3, and contents of scratch files are summarised in Table 1.

From Fig. 3 it may be seen that integrals of kernel-shape function products appearing in equation (13) are evaluated only once. No matrix inversions are carried out, it being more economical to factorise into lower and upper triangular matrices. The matrix **A** of equation (14) is constructed and factorised only once. The matrix C_{11} of equation (16) is generally much smaller than the matrix A, so the cost of constructing and factorising it once during each iteration is usually insignificant. The most expensive operation is the Newton-Raphson iteration for the phreatic surface performed once per iteration. To reduce the cost of this iteration, the location of the (n-1)th phreatic surface is taken as the initial estimate when locating the nth surface. For interior points x near S, substitution of the appropriate parametric representations and use of Gaussian quadrature formulae in equations (4) and (5) does not give accurate results; therefore, when $u(x)$ and $v_i(x)$ are required for a point x near S, their values at a point further from S are calculated and the desired results obtained by linear interpolation between that point and the nearest point to x on S. This procedure incurs the inconvenience and overhead of locating the nearest surface point to x, but the routines developed for this purpose are reliable and of efficiency such that the overhead is small. In GFLOW, interpolation is carried out if x is nearer to S than about 0.7 times the length of the nearest boundary element. If the orders of Gaussian formulae were chosen according to estimated rapidity of variation of the integrand [10], then this distance, and with it the errors incurred by linear interpolation, could be reduced.

Example of numerical modelling

Consider an exploration tunnel with a horse-shoe shaped cross-section, excavated in a jointed rock mass and continuously drained by pumping (see Figs. 4 and 5). The tunnel is 4m wide and 4m high, and its floor is 80m below ground level. There is recharge at ground level at distances greater than 400m from the centreline of the tunnel. A 1000m x 300m cross-section of rock is modelled, the outer boundary being represented by 16 elements and that of the tunnel by 12 as shown in Figs. 4 and 5. There are vertical and horizontal joint sets, such that the permeabilities of the equivalent continuum are 5.2×10^{-4}m/sec and 3.78×10^{-5}m/sec in the vertical and horizontal directions respectively. The computed inflow, after 10 iterations, is 0.0039m^3/sec per metre run of tunnel. As may be seen from Figs. 4 and 5, the phreatic surface is computed to draw down to the crown of the tunnel.

In this example, convergence of the iteration was slow because at points on the estimated phreatic surface near the tunnel, large adjustments of inflow at ground level were required to zeroise the flow $s^{(n-1)}(y)$ (see eqn (9)) across the estimated phreatic surface. It is intended to modify the iteration so that adjustments of inflow at nodes on S' are computed by solving a system of simultaneous equations of order s (see eqn (14)), rather than by solving a system of order r for adjustments on $S_w^{(n-1)}$ and computing adjustments on $S_d^{(n-1)}$ according to eqn (9) as at present. Flow across the estimated phreatic surface near an underground opening will then be zeroised largely by adjustments of inflow at nodes on the boundary of the opening, rather than at nodes at ground level.

CONCLUSIONS

It has been demonstrated that the boundary equation method can be used to solve problems of unconfined flow in porous media, but as indicated in the preceding section, the iteration for the phreatic surface must be modified to ensure that it converges reliably. There were two motives for developing an algorithm in which the boundary element mesh is not redefined at each iteration: to reduce computing cost, and to ensure that the computation could be allowed to proceed to a conclusion without the need to check periodically whether the problem of approaching or intersecting boundaries is likely to be encountered and take avoidance action where necessary. In practice, a system of simultaneous equations must be solved at each iteration, and the computing cost may in fact be comparable with that of an algorithm in which the boundary element mesh is redefined. However, the algorithm described here is free of the problems posed by intersecting boundaries, and once the iteration is modified will operate reliably without user intervention.

The applicability of the boundary element method to problems of groundwater flow through rock depends upon the validity of the representation of the jointed rock mass by an equivalent continuum. In future developments, the existing numerical model should be interfaced with finite elements which will represent major discontinuities, these being discontinuities of relatively high permeability, the dimensions of which are of the same order as those of the mass of rock under consideration.

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File	Contents	File	Contents
LIN	integrals of kernel-shape function products	LWM	upper triangular factor of C_{11}
LUM	upper triangular factor of matrix A	LWL	lower triangular factor of C_{11}
LUL	lower triangular factor of matrix A		
LIM	matrix $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$		

Table 1: Scratch files created by GFLOW

List of Figures

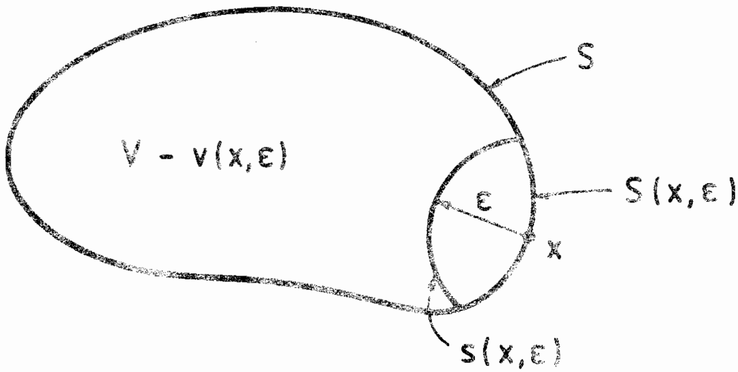


Figure 1: Region for which the reciprocal theorem in written

1 Ábra A tartomány, amelyre a reciprok tételt felírják

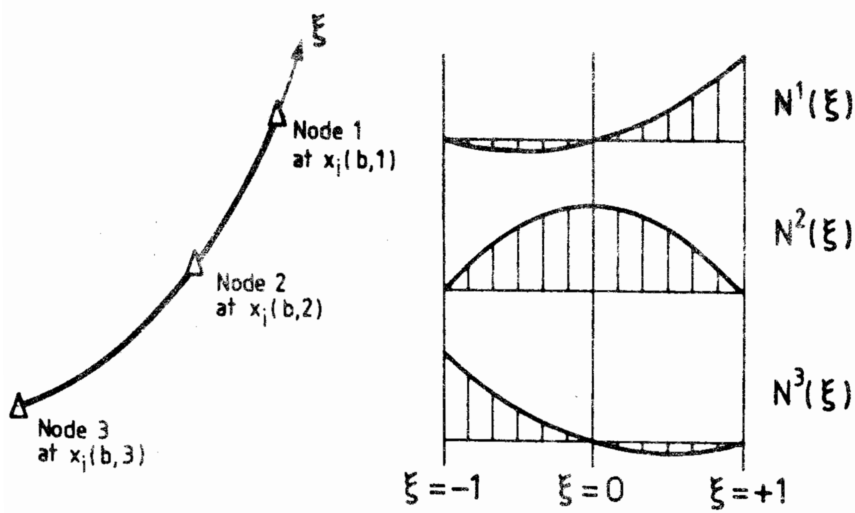


Figure 2: Boundary element S_b and shape functions.

2 Ábra A határfelület elemei, S_b és az alak függvények

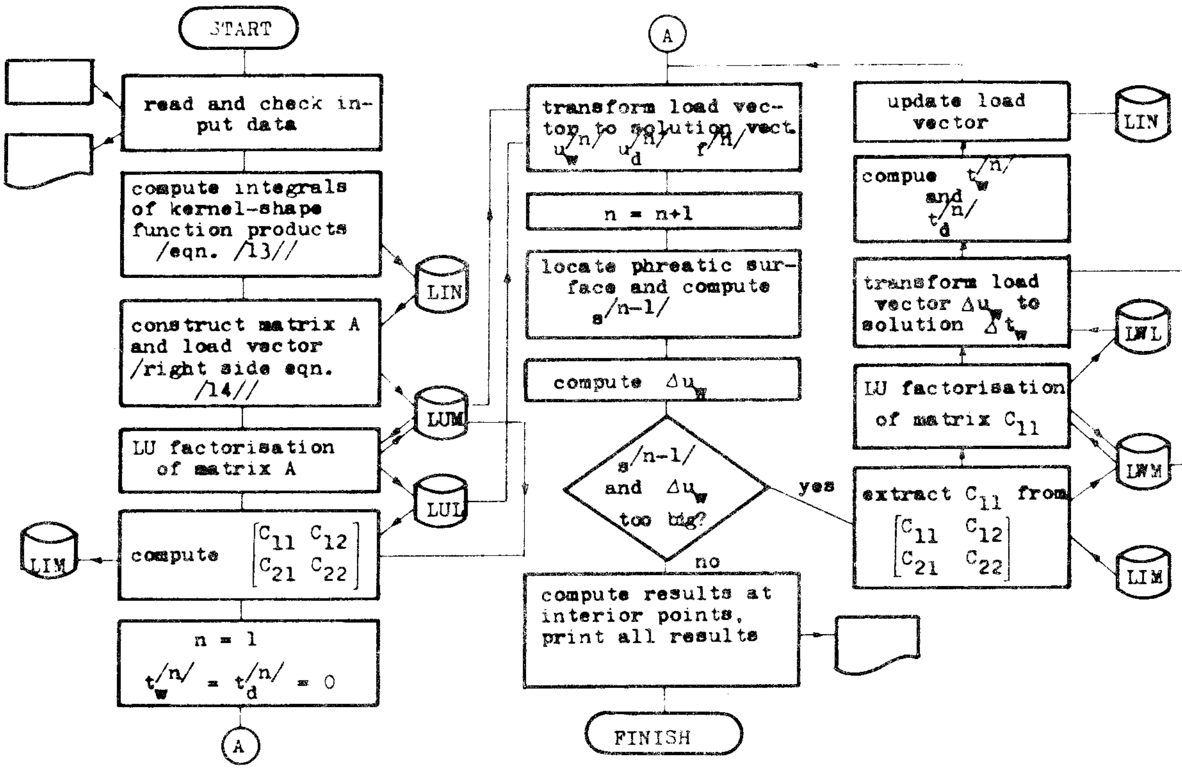

START
read and check input data
compute integrals of kernel-shape function products /eqn. /13//
construct matrix A and load vector /right side eqn. /14//
LU factorisation of matrix A
compute $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$
n = 1
$t_w^{/n/} = t_d^{/n/} = 0$
A
LIN
LUM
LUL
LIM
A
transform load vector to solution vect. $u_w^{/n/}$ $u_d^{/n/}$ $f^{/n/}$
n = n+1
locate phreatic surface and compute $s^{/n-1/}$
compute Δu_w
$s^{/n-1/}$ and Δu_w too big?
yes
no
compute results at interior points, print all results
FINISH
update load vector
compue $t_w^{/n/}$ and $t_d^{/n/}$
transform load vector Δu_w to solution Δt_w
LU factorisation of matrix C_{11}
extract C_{11} from $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$
LIN
LWL
LWM
LIM

Figure 3.: Simplified flow chart of program GFLOW

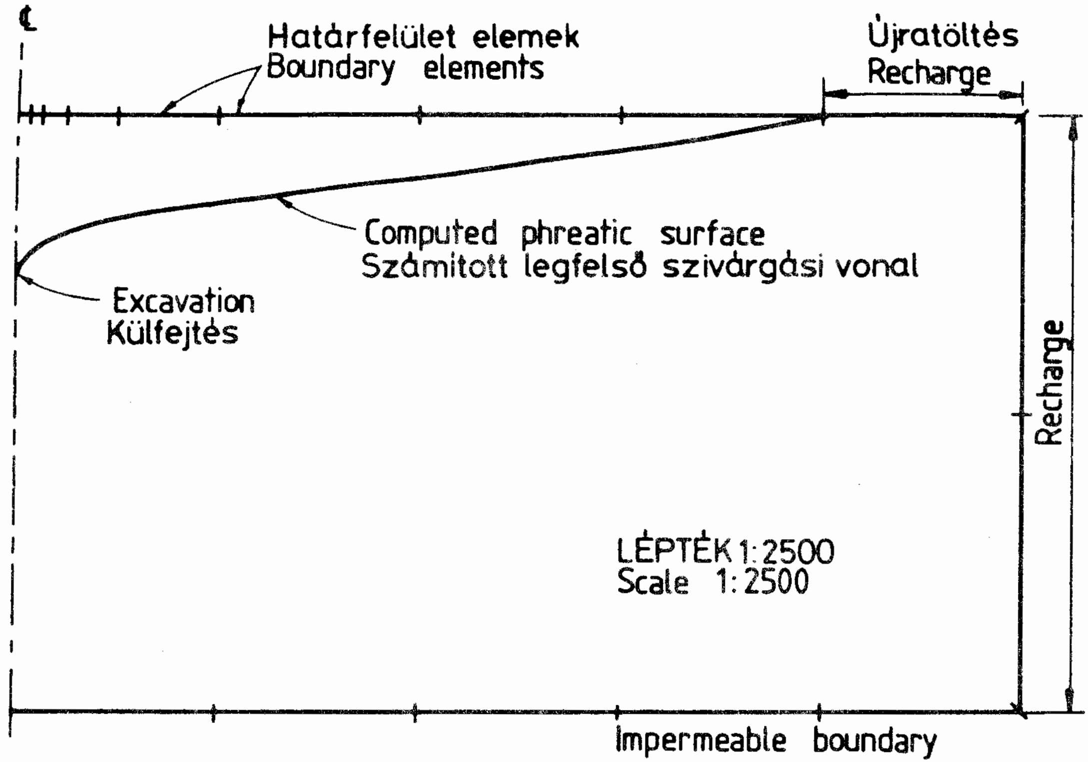
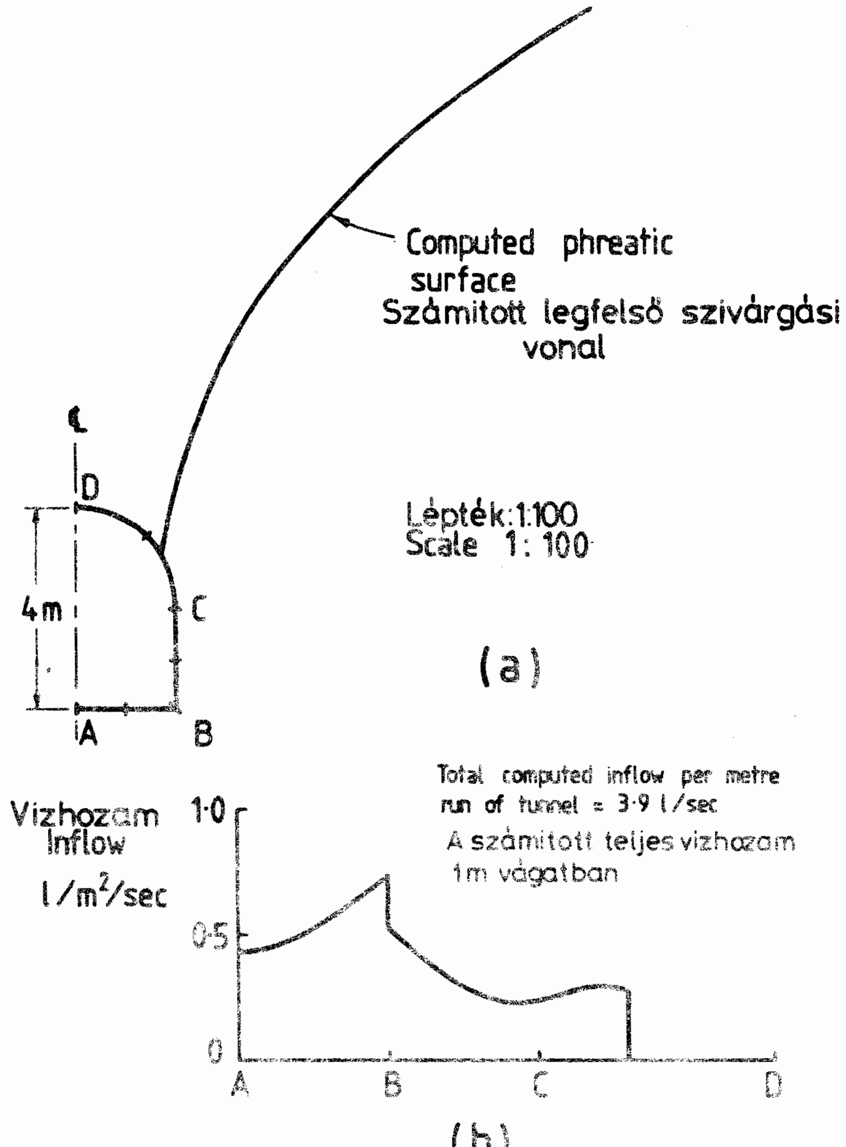


Figure 4: Boundary element mesh and computed phreatic surface.
4 ábra Határfelület rácspontjai és a számított legfelső szivárgási vonal



a) A vágat környezetében számitott legfelső szivárgási vonal

Figure 5: a) Computed phreatic surface in the region of the tunnel, and b) computed inflow across the tunnel boundary

5 Ábra b) a vágat felületén számitott vizhozam